

# Proceedings of “Bianisotropics’93”

*Seminar on electrodynamics of chiral and bianisotropic media*

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12–14 October 1993

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Gomel, Belarus

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**Ari Sihvola, Sergei Tretyakov, Igor Semchenko (editors)**



Helsinki University of Technology  
Faculty of Electrical Engineering  
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Report 15

# Proceedings of "Bianisotropics'93"

*International seminar on  
electrodynamics of chiral and bianisotropic media*

Ari Sihvola, Sergei Tretyakov, Igor Semchenko (editors)

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## **Abstract**

The international seminar *Bianisotropics'93* was held at the University of Gomel in southeastern Belarus in 12 – 14 October 1993. The topic of the meeting was the electromagnetics of complex materials, like, for example chiral, nonreciprocal, gyrotropic, and anisotropic media. This Proceedings contains written reports of the presentations in *Bianisotropics'93* as well as contact information of the participants of the seminar.

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## Introduction

This book contains the proceedings of *Bianisotropics'93*, an international seminar on electrodynamics of chiral and bianisotropic media. The seminar was held 12 – 14 October 1993, in the city of Gomel, in the southeastern part of the republic of Belarus. The seminar was organised as a continuation of *Bi-isotropics'93*, which was a workshop on novel microwave materials, held in Espoo, Finland, in February 1993 (for the scientific contributions of *Bi-isotropics'93*, see No. 137 of this Report Series).

The venue, why Gomel? — Since 1950's, the study of electromagnetics and optics of gyrotropic and bianisotropic materials has been intense in Belarus, and in the former Soviet Union in general. Academician F.I. Fedorov, his research groups, and his students have created a fertile basis for research on electrodynamics of complex materials. No wonder therefore, that the Gomel seminar attracted scientists and engineers in the international scale. The affiliations of the 30 participants of *Bianisotropics'93* can be found in this report.

The seminar put a strong emphasis on the scattering from helical structures, which is a key issue in the chiral and anisotropic electromagnetics research. This aspect can be also seen in the reports of the Proceedings. The Proceedings reflects the Gomel seminar quite fully; only two of the presentations given in *Bianisotropics'93* are missing from this report (those of A. Serdyukov and V. Shepelevich). On the other hand, the paper by V. Semenenko and D. Ryabov on page 116 has been included although it was not presented in Gomel.

While one of the objectives of the present Proceedings also is to promote contacts across the former iron curtain, the phone, fax, and electronic mail codes are listed for the participants. Through these means, the speed and efficiency of communications between the scientists in East and West has increased (and is increasing) dramatically in the present times. Catch the opportunity and contribute to the global warming! Technical and scientific interaction is to be fostered.

*Bianisotropics'93* has been supported by a few institutions. We acknowledge the generous help from

- Regional Council of Gomel
- Gomel State University
- VVV Company (Production-Commercial Company of Gomel. Automobiles, Computers, Software. Phone/fax (7 0232) 579-700, 579-750, 573-793)
- The Electromagnetics Laboratory of Helsinki University of Technology
- IEEE (The Institute of Electrical and Electronics Engineers) MTT (Microwave Theory and Techniques) Society within Region 8

And the continuation? Demand for seminars on complex media electromagnetics is great. The next workshop will be held in France, in 16 – 20 May, 1994. The organising institution is the French Atomic Energy Commission CEA-CESTA. For more information, please contact F. Mariotte (see list of participants).

A.S., S.T., I.S.  
Espoo, St. Petersburg, Gomel  
November 1993

— *Bianisotropics '93* —

**Electrodynamics of Chiral and Bianisotropic Media**  
**International Seminar in Gomel, Byelarus, 12–14 October 1993**

**ORGANISING COMMITTEE**

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CHAIRMAN: Professor A. Serdyukov (*Gomel, Belarus*)  
MEMBERS: Dr. I. Semchenko (*Gomel, Belarus*)  
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**Electrodynamics of Chiral and Bianisotropic Media**  
International Seminar in Gomel, Byelarus, 12–14 October 1993

**FINAL PROGRAMME**

**Tuesday, 12 October**

Morning Session. *Chairman – F.I. Fedorov (Belarus)*

1. F.I. FEDOROV (Academy of Sciences, Minsk, Belarus)  
Opening of the Workshop
2. L.A. SHEMETKOV (President, Gomel State University, Belarus)  
Welcoming Address
3. A.H. SIHVOLA (Helsinki University of Technology, Finland)  
*Similarities and differences between bi-isotropic and anisotropic electromagnetic quantities*
4. A.P. VINOGRADOV (Scientific Center for Applied Problems in Electrodynamics, Moscow, Russia)  
*Calculation of electromagnetic parameters of chiral systems*
5. F. MARIOTTE (French Atomic Energy Commission, Le Barp, France)  
*Backscattering of the thin wire helix: Analytical model, numerical study and free-space measurements. Application to chiral composite modelling*

Afternoon Session. *Chairman – S.A. Tretyakov (Russia)*

6. M.V. KOSTIN, V.V. SHEVCHENKO (Institute of Radioengineering and Electronics, Moscow, Russia)  
*Artificial magnetics based on circular film elements*
7. G.A. KRAFTMAKHER (Institute of Radioengineering and Electronics, Moscow, Russia)  
*Abnormally high electromagnetic activity in chiral media magnetically inactive at low frequencies*
8. L. ARNAUT (University of Manchester Institute of Science and Technology, United Kingdom)  
*Optimal polarisations of a general biisotropic half-space*

9. S.A. TRETYAKOV, A.A. SOCHAVA (St. Petersburg State Technical University, Russia)

*Plane electromagnetic waves in uniaxial bianisotropic media*

10. F. GUÉRIN (Thomson-CSF, Central Research Laboratories, France)

*Energy dissipation and absorption in biisotropic media*

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## Wednesday, 13 October

*Chairman – A.H. Sihvola (Finland)*

11. S. BOLIOLI (ONERA-CERT, Toulouse, France)

*How to tailor and orient metallic helices to get microwave chirality*

12. G.N. BORZDOV (Belarus State University, Minsk, Belarus)

*Lorentz-covariant solution of the inverse problem of reflection and transmission for a dispersive bianisotropic medium*

13. L. ARNAUT (University of Manchester Institute of Science and Technology, United Kingdom)

*Power reflection and absorption for lossy chiral media*

14. V.N. BELYI, G.V. KULAK (Institute of Physics, Minsk, Belarus)

*The interaction of electromagnetic and acoustic waves in bianisotropic crystals with electrically induced anisotropy*

15. F. GUÉRIN (Thomson-CSF Central Research Laboratories, France)

*Experimental aspects of microwave chirality research*

## Poster Session

16. E.A. EVDISCHENKO, A.F. KONSTANTINOVA, B.N. GRECHUSHNIKOV (Institute of Crystallography, Moscow, Russia)

*The composite gyrotropic plates*

17. I.V. SEMCHENKO, B.B. SEVRUK, S.A. KHAKHOMOV (Gomel State University, Belarus)

*Acousto-electron interaction in conducting crystals of ferroelectric ceramic in the condition of inducing of piezoelectric, anisotropic and gyrotropic properties by the rotating electric field*

18. G.S. MITYURICH, E.G. STARODUBTSEV (Gomel State University, Belarus)

*Dissipative properties of gyrotropic superlattices in the long-wavelength approximation*

19. S.S. GIRGEL, T.V. DEMIDOVA (Gomel State University, Belarus)  
*Symmetry of tensors and optical properties of directions in magnetic crystals*

20. S.S. GIRGEL, T.V. DEMIDOVA (Gomel State University, Belarus)  
*Bianisotropies of quasicrystals. Symmetry aspects*

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## Thursday, 14 October

Morning Session. *Chairman – S.A. Tretyakov (Russia)*

21. F.I. FEDOROV, (Academy of Sciences, Minsk, Belarus)  
*On general wave normal equation for bianisotropic media*

22. A.F. KONSTANTINOVA, E.A. EVDISCHENKO, B.N. GRECHUSHNIKOV  
(Institute of Crystallography, Moscow, Russia)  
*The determination of optical anisotropic parameters of absorbing gyrotropic media*

23. V.V. SHEPELEVICH (Mozyr Pedagogical Institute, Belarus)  
*The effect of gyrotropy on light wave interaction in cubic piezocrystals*

24. A.N. SERDYUKOV (Gomel State University, Belarus)  
*Electromagnetic waves in Relativistic Gravity Theory*

25. I.N. AKHRAMENKO, I.V. SEMCHENKO (Gomel State University, Belarus)  
*Non-collinear interaction of electromagnetic waves in gyrotropic crystals with rapidly rotating anisotropy*

26. A.N. SERDYUKOV and F.I. FEDOROV  
*Closing of the Workshop*

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The following is a list of recent publications that discuss the topic of our Workshop. This list complements the earlier bibliography, given in the Proceedings of the February 1993 workshop 'Bi-isotropics'93'.

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# Similarities and differences between bi-isotropic and anisotropic electromagnetic quantities

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**ABSTRACT:**— To choose a well-balanced topic for a presentation in a workshop named *Bianisotropics'93*, this study focuses on the common elements in the electromagnetic behavior of two distinct material classes: bi-isotropic and anisotropic media. Bi-isotropic media are insensitive in their responses to the direction of the field vectors, whereas anisotropic media do not react electrically to magnetic excitation and vice versa. The polarizability matrix elements of a small homogeneous sphere depends on the material parameters, and this presentation studies the analogies between the polarizability components of the two different material spheres.

The well-known [1] bi-isotropic constitutive relations between the electric ( $\vec{E}$ ,  $\vec{D}$ ) and magnetic ( $\vec{H}$ ,  $\vec{B}$ ) vectors

$$\vec{D} = \epsilon \vec{E} + (\chi - j\kappa) \sqrt{\mu_o \epsilon_o} \vec{H} \quad (1)$$

$$\vec{B} = \mu \vec{H} + (\chi + j\kappa) \sqrt{\mu_o \epsilon_o} \vec{E} \quad (2)$$

are followed in the present analysis. If one wishes to consider the electromagnetical reaction of a small inclusion of bi-isotropic material — characterized by the material parameters  $\epsilon$  (permittivity),  $\mu$  (permeability),  $\kappa$  (chirality),  $\chi$  (nonreciprocity), — the electric and magnetic dipole moments  $\vec{p}_e$ ,  $\vec{p}_m$  need to be evaluated. The response is contained in the polarizability components of the inclusion, which can be arranged in a  $2 \times 2$  matrix form:

$$\begin{pmatrix} \vec{p}_e \\ \vec{p}_m \end{pmatrix} = \begin{pmatrix} \alpha_{ee} & \alpha_{em} \\ \alpha_{me} & \alpha_{mm} \end{pmatrix} \begin{pmatrix} \vec{E} \\ \vec{H} \end{pmatrix} \quad (3)$$

In the polarizability symbols  $\alpha_{ij}$ , there are two indices: the first ( $i$ ) denotes the polarization type, and the second ( $j$ ) is for the origin of the polarization.

The polarizabilities of a sphere with volume  $V = 4\pi a^3/3$  are

$$\alpha_{ee} = 3\epsilon_o V \frac{(\epsilon - \epsilon_o)(\mu + 2\mu_o) - (\chi^2 + \kappa^2)\mu_o\epsilon_o}{(\mu + 2\mu_o)(\epsilon + 2\epsilon_o) - (\chi^2 + \kappa^2)\mu_o\epsilon_o} \quad (4)$$

$$\alpha_{em} = 3\mu_o\epsilon_o V \frac{3(\chi - j\kappa)\sqrt{\mu_o\epsilon_o}}{(\mu + 2\mu_o)(\epsilon + 2\epsilon_o) - (\chi^2 + \kappa^2)\mu_o\epsilon_o} \quad (5)$$

$$\alpha_{me} = 3\mu_o\epsilon_o V \frac{3(\chi + j\kappa)\sqrt{\mu_o\epsilon_o}}{(\mu + 2\mu_o)(\epsilon + 2\epsilon_o) - (\chi^2 + \kappa^2)\mu_o\epsilon_o} \quad (6)$$

$$\alpha_{mm} = 3\mu_o V \frac{(\mu - \mu_o)(\epsilon + 2\epsilon_o) - (\chi^2 + \kappa^2)\mu_o\epsilon_o}{(\mu + 2\mu_o)(\epsilon + 2\epsilon_o) - (\chi^2 + \kappa^2)\mu_o\epsilon_o} \quad (7)$$

These polarizabilities are very essential in the modeling of bi-isotropic media. For the nonchiral reciprocal limit  $\kappa \rightarrow 0$ ,  $\chi \rightarrow 0$ , the polarizabilities simplify to the well-known expressions:

$$\alpha_{ee} = 3\epsilon_o V \frac{\epsilon - \epsilon_o}{\epsilon + 2\epsilon_o} \quad (8)$$

$$\alpha_{mm} = 3\mu_o V \frac{\mu - \mu_o}{\mu + 2\mu_o} \quad (9)$$

$$\alpha_{me} = \alpha_{em} = 0 \quad (10)$$

Note here the decoupling of the electric and magnetic quantities, compared with the bi-isotropic case.

Let us next take a look at the polarizability behavior of *anisotropic* small inclusions. Bi-isotropy referred to isotropy in the sense that no direction in the space is special, the macroscopic electromagnetic behavior of the material is independent of spatial rotations. However, magnetoelectric coupling exists. In contrast, anisotropy accounts only for electric polarization due to electric excitation<sup>1</sup> but the response is sensitive to the vector directions.

Dielectric anisotropy means that the polarization caused by electric field is generally not in the same direction as the field itself. Correspondingly, in anisotropic magnetic materials, the average magnetic dipole moment density is only in principal axes directions parallel to the magnetic field. In bi-isotropic media, on the other hand, there are no special axes, or directions. Therefore, it may seem strange that so different polarization mechanisms as in these two different classes of materials, there exist similar laws in the polarizability descriptions.

The constitutive relations of anisotropic media are formally simpler than bi-isotropic ones. For dielectrically anisotropic media (for example magnetoplasma), the permittivity is dyadic:

$$\vec{D} = \vec{\epsilon} \cdot \vec{E} \quad (11)$$

and for magnetically anisotropic media (for example, ferrites), permeability is dyadic:

$$\vec{B} = \vec{\mu} \cdot \vec{H} \quad (12)$$

<sup>1</sup>Or, in the magnetically anisotropic case, magnetic polarization due to incident magnetic field.

Let us consider an anisotropic sphere, with permittivity dyadic

$$\bar{\epsilon} = \epsilon_x \bar{u}_x \bar{u}_x + \epsilon_y \bar{u}_y \bar{u}_y + \epsilon_z \bar{u}_z \bar{u}_z + \epsilon_g \bar{u}_z \times \bar{I} \quad (13)$$

This consists of a symmetric (biaxial) part and antisymmetric (gyrotropic) part. The gyrotropy axis is here assumed to be aligned with one of the symmetry axes  $\bar{u}_z$ .  $\epsilon_g$  is the measure of gyrotropy. The gyrotropic character of the permeability in ferrites can be exploited in nonreciprocal microwave applications, like circulators and isolators, but also nonreciprocal permittivities are being studied for the use of gyroelectric waveguides [2].

The polarizability dyadic of this sphere can be shown to be [3]

$$\bar{\alpha} = \sum_{i,j=x,y,z} \alpha_{ij} \bar{u}_i \bar{u}_j \quad (14)$$

with components

$$\begin{aligned} \alpha_{xx} &= 3\epsilon_o V \frac{(\epsilon_x - \epsilon_o)(\epsilon_y + 2\epsilon_o) + \epsilon_g^2}{(\epsilon_x + 2\epsilon_o)(\epsilon_y + 2\epsilon_o) + \epsilon_g^2} \\ \alpha_{yy} &= 3\epsilon_o V \frac{(\epsilon_y - \epsilon_o)(\epsilon_x + 2\epsilon_o) + \epsilon_g^2}{(\epsilon_x + 2\epsilon_o)(\epsilon_y + 2\epsilon_o) + \epsilon_g^2} \\ \alpha_{zz} &= 3\epsilon_o V \frac{\epsilon_z - \epsilon_o}{\epsilon_z + 2\epsilon_o} \\ \alpha_{xy} &= -\alpha_{yx} = 3\epsilon_o V \frac{-3\epsilon_o \epsilon_g}{(\epsilon_x + 2\epsilon_o)(\epsilon_y + 2\epsilon_o) + \epsilon_g^2} \\ \alpha_{zz} &= \alpha_{zx} = \alpha_{yz} = \alpha_{zy} = 0 \end{aligned} \quad (15)$$

There are striking similarities as one compares the gyrotropic polarizability components of (15) to the polarizability matrix of a bi-isotropic sphere (4) - (7).

In (15) the gyrotropy parameter  $\epsilon_g$  affects the polarizability components. If it vanishes, the matrix becomes diagonal and the components become simple functions of the permittivities like in the perfect isotropic case. However, in the gyrotropic case, there is one component that is not affected by  $\epsilon_g$ . This is the z-directed copolarizability  $\alpha_{zz}$  which is the same as isotropic. It means that, for example in the case of a ferrite sphere, the gyrotropy has no effect on the copolarizability in the external magnetic field direction.

On the other hand, gyrotropy affects the transversal components  $\alpha_{xx}$  and  $\alpha_{yy}$  as also the off-diagonal components  $\alpha_{xy}$  and  $\alpha_{yx}$ . Here  $\epsilon_g$  plays a similar role as the chirality parameter  $\kappa$  or nonreciprocity parameter  $\chi$  in the case of bi-isotropic sphere. However, there is a change of sign: the denominator of the gyrotropic case is

$$(\epsilon_x + 2\epsilon_o)(\epsilon_y + 2\epsilon_o) + \epsilon_g^2$$

whereas the corresponding expression for the chiral (Pasteur) case is

$$(\epsilon + 2\epsilon_o)(\mu + 2\mu_o) - \kappa^2 \mu_o \epsilon_o$$

and in the nonreciprocal (Tellegen) case

$$(\epsilon + 2\epsilon_o)(\mu + 2\mu_o) - \chi^2\mu_o\epsilon_o$$

and in the general bi-isotropic case

$$(\epsilon + 2\epsilon_o)(\mu + 2\mu_o) - (\chi^2 + \kappa^2)\mu_o\epsilon_o.$$

These quantities, and also the polarizability expressions, have full correspondence if the gyrotropy is imaginary:  $\epsilon_g = jg$  where  $g$  is real. This is in fact the case in magnetoplasma [4, 5] or in the case of the permeability of ferrites [6]:

$$\bar{\bar{\mu}} = \mu_a \bar{u}_z \bar{u}_z + \mu_t (\bar{\bar{I}} - \bar{u}_z \bar{u}_z) + jg \bar{u}_z \times \bar{\bar{I}} \quad (16)$$

The imaginary nature of the gyrotropy in the permittivity/permeability expression completes the analogy with respect to bi-isotropic media. We can write the following correspondence table:

Pasteur medium	Tellegen medium	Dielectrically gyrotropic	Magnetically gyrotropic
$\kappa$	$\chi$	$j\epsilon_g$	$j\mu_g$
$\epsilon$	$\epsilon$	$\epsilon_x$	$\mu_x$
$\mu$	$\mu$	$\epsilon_y$	$\mu_y$
$\alpha_{ee}$	$\alpha_{ee}$	$\alpha_{xx}$	$\alpha_{xx}$
$\alpha_{mm}$	$\alpha_{mm}$	$\alpha_{yy}$	$\alpha_{yy}$
$\alpha_{em}$	$\alpha_{em}$	$\alpha_{xy}$	$\alpha_{xy}$
$\alpha_{me}$	$-\alpha_{me}$	$\alpha_{yx}$	$\alpha_{yx}$

Due to this wonderful correspondence, all the conclusions and numerical results that have been made for bi-isotropic media are valid (*mutatis mutandis*) for spheres made of gyrotropically anisotropic material, regardless of the nature (be it of dielectric or magnetic origin) of the gyrotropy.

## POSTSCRIPT

During the talk and in the discussion session of Gomel Workshop on the topic of this paper, the issue about the meaning of the very term *gyrotropy* was raised. It turned out that scientists understand this concept differently, even those within the same country and research culture. Therefore also the use of the term is by no means precise, especially as people from separate fields talk about these problems.

In the most vague sense, gyrotropy means any deviation from the simple isotropic behaviour  $\bar{D} = \epsilon \bar{E}$ ,  $\bar{B} = \mu \bar{H}$ . This approach would embrace all anisotropy and bi-isotropy within the domain of gyrotropy. However, as the present paper attempts to show, the aspects of nonisotropic phenomena in electric, magnetic, and magnetoelectric media can be classified more concisely.

"Gyrotropy" originates — again — from Greek; *gyros* meaning 'ring' or 'round' ( $\gammaυρος$ ). Hence the most natural meaning for gyrotropy is to affiliate it with those media that are characterised by the property of rotating the polarisation plane of the linearly polarised electromagnetic wave. Ferrites and magnetoplasma are therefore gyrotropic, and these materials also possess the permittivity (or permeability) dyadic, which is a *gyrotropic dyadic* (i.e. the dyadic contains a component of  $\bar{u} \times \bar{I}$ ).

This is not the case for Pasteur media, which also rotate the polarisation plane, but the material parameters are isotropic, and the parameter dyadics multiples of the unit dyadic. Still, due to the rotation, chiral media deserve the label of gyrotropy, although the medium is different (it is reciprocal and isotropic, instead of nonreciprocal and anisotropic, like ferrites).

If we keep the criterium of rotation for gyrotropy, we have both reciprocal gyrotropy and nonreciprocal gyrotropy. But Tellegen medium is not gyrotropic, although it is magnetoelectric. Also one can find examples of more complicated bianisotropic media which are not gyrotropic, although these contain magnetoelectric coupling which is of a dyadic form, like so called special  $\Omega$  media discussed in other talks of the Gomel Workshop, and also in the present Proceedings.

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# Microscopic properties of a chiral object

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The most part of investigations concerning the propagation of electromagnetic waves through inhomogeneous media deal with the cases when any scale of inhomogeneity  $a$  is small in terms of both free space wavelength  $\lambda_0$  and wavelength  $\lambda_{int}$  inside any domain of the medium ( $ak \ll 1$ ), where  $k$  is a wavenumber. In this case it is possible to consider the interaction of an electromagnetic wave with inclusions in steady state approximation ( $ak = 0$ ) and successfully use some kind of EMA theory [1,2]. Recently, the attention of scientists has been attracted to cases when wavelength  $\lambda_{int}$  may be comparable with some characteristic length  $a$  of inhomogeneity [3, 4, 5]. In this case, one can not neglect the value of  $(ak)$  and has to take into account retardation and nonpotential (vortex) character of fields. The exact consideration implies introducing nonlocal constitutive equations (see [3, 6]). Fortunately if  $(ak) < 1$  in matrix material it is still reasonable to work with local constitutive equations. It is the case of high conducting inclusions. Inside the inclusions  $(ak) > 1$ ; fields here are vortex, and currents flow only in thin skin layer. Outside the inclusions the value of  $k$  may be so small that  $(ak) < 1$ . To take into account nonpotential feature of fields it is sufficient to use renormalized value of conductivity and introduce an effective permeability due to eddy currents [2, 4, 5]. Both of these effects are of the second order in  $(ak)$ .

Now we consider chirality that is an effect of the first order in  $(ak)$ . We study composite material containing wire helix inclusions. We confine ourselves to the case when the external sizes of the wire helix: the length  $L$  of the helix, the wire radius  $r_w$ , and the helix radius  $r_h$  are less than the wavelength in surrounding medium. We hope that subject to  $(kL) < 1$  it is possible to describe the system by local constitutive equations with additional terms. Nevertheless, the effects may be significant because the total length  $\mathcal{L}$  of the wire may be about  $\lambda/2$ ; and a resonance may appear.

The absence of the center of symmetry in the helix yields the rotation of polarization of scattered fields [7]. Let us consider an electromagnetic wave that falls on the helix along axis  $z$  with electric field polarized parallel to  $y$ -axis. The field causes the movement of charges up and down the helix. The current flowing along  $y$ -axis will emit  $y$ -polarized wave. There also exists  $x$ -component of the current which might radiate  $x$ -polarized wave. Since in opposite parts of a helix turn the  $x$ -currents flow in opposite directions the radiation of  $x$ -polarized wave is due to retardation ( $ka$ )

of the electro-magnetic wave. Thus there appears  $x$ -component of electric field that means rotation of polarization and this rotation is proportional to  $(ka)$ .

Below we consider this effect quantitatively for a single helix inclusion. We confine our consideration to calculating the magnitude of induced electric and magnetic moments as well as chiral factor of single inclusion keeping in mind that there are many articles [8, 9] devoted to the problem of cooperative interaction of chiral inclusions.

We shall deal with frequency domain representation. Complex notation is used throughout the article with  $e^{i\omega t}$  time dependence assumed, and then suppressed.

If we deal with perfectly conducting thin wire we can neglect the angular currents and consider the tangent components only. In this case it is more convenient to deal with the whole current  $J$  and linear charge density  $\rho$  obtained by integration current and charge densities over the cross section. We shall also use the following approximation for Green function [10]:

$$G(s, s') = \left\{ \frac{e^{-ik[(x_r - x_{r'})^2 + (y_r - y_{r'})^2 + (z_r - z_{r'})^2 + r_w^2]^{0.5}}}{[(x_r - x_{r'})^2 + (y_r - y_{r'})^2 + (z_r - z_{r'})^2 + r_w^2]^{0.5}} \right\} \quad (1)$$

where  $s$  is an arc length at the point  $r = \{x_r, y_r, z_r\}$ .

For the tangent to the wire component of the electric field on the helix to vanish, it is require that  $E_s^{total} = E_s^{inc} + E_s^* = 0$ , where  $E_s^{inc}$  and  $E_s^*$  are tangent components of the incident and scattered electric field. This leads to the first order Fredholm integral equation which is incorrect [11]. The approximation (1) will not lead us to the correct solution unless special precautions are taken. The correct results may be obtained in scheme proposed by Mei [12].

Using the aforementioned technique we numerically solve the diffraction problem and obtain the relationship

$$J(s)l_i(s) = i\omega \int_L A_{ij}(s, s') E_j^{inc}(s') ds' = \omega T_{ij}(s) E_{0j} \quad (2)$$

Following the scheme suggested by Born [7] it is possible to evaluate the electric moment  $P$ , the magnetic moment  $M$ , and the chiral factor  $\beta$  for a single inclusion.

The value of the  $P$  we obtain using the law of charge conservation:  $i\omega\rho = -\text{div}\vec{j}$ . For thin wire we rewrite it in the form  $\rho = \frac{-1}{i\omega\pi r_w^2} \frac{dJ(s)}{ds}$  and obtain:

$$\begin{aligned} P_i e^{-ik\langle r \rangle} &= \int_L r_i(s) \rho(s) (\pi r_w^2) \frac{ds}{V} = \\ &= \frac{i}{\omega V} \int_L r_i(s) \frac{dJ(s)}{ds} ds = \frac{i}{\omega V} r_i J \Big|_0^L - \frac{i}{\omega V} \int_L J(s) l_i(s) ds \end{aligned}$$

Using zero boundary conditions for current we obtain electric moment per unit volume:

$$P_i = \frac{1}{V} \int_L \int_L A_{ij}(s, s') e^{-ik_1(r_i - \langle r_i \rangle)} ds ds' E_j^0 = a_{ij} E_j^0 \quad (3)$$



The effects considered here are beyond the scope of steady state approximation, there is no reason to think that  $a_{ij} = a_{ji}$ . Below we focus our attention on an antisymmetrical part of the susceptibility tensor. At first, we give some speculations favoring the following form of antisymmetrical part:  $a_{ij}^a = k_m \alpha_{ij,m}$ .

Since we are interested in first-order effect in  $(ak)$  we can use the following expansion of the incident field:

$$E_j^{inc}(s) = E_{0j} e^{-i(k_m x_m)} = E_j^{(0)inc} + E_j^{(1)inc} = E_{0j} - i(k_m x_m) E_{0j} \quad (4)$$

that yields the corresponding modification of (2):

$$J^{(1)}(s) l_i(s) = \omega k_m \int_L A_{ij}(s, s') x_m(s') E_{0j} ds' = \omega T_{ij,m}^{(1)} k_m E_{0j}$$

For antisymmetrical part of the susceptibility tensor we obtain:

$$\begin{aligned} a_{ij}^a &= \int_L \int_L \frac{1}{2V} [A_{ij}(s, s') - A_{ji}(s, s')] (-ik_m x_m) ds ds' = \\ &= -ik_m \int_L \frac{1}{2V} [T_{ij,m}^{(1)}(s) - T_{ji,m}^{(1)}(s)] ds = k_m \alpha_{ij,m} \end{aligned} \quad (5)$$

The zero-order term of  $a_{ij}^a$  is zero because it corresponds to the case of steady state fields ( $ka = 0$ ) and for this case permittivity tensor is symmetrical.

Following the common procedure [7] let us introduce a tensor  $g_{ij}$ :

$$g_{xj} = i\alpha_{yz,j} = -i\alpha_{zy,j} \quad j = 1, 2, 3; \text{ with cyclic transposition of } \{x, y, z\}$$

and write  $P$  of the form  $P_i = a_{ij}^* E_{0j}^{inc} + ie_{ijk} \beta_j E_{0k}^{inc}$  where  $\beta_j = g_{jl} k_l$ .

If we dealing with isotropic materials averaging over the angles produces  $g_{ij} = g\delta_{ij}$  with  $g = (g_{11} + g_{22} + g_{33})/3$ . Using  $\vec{\beta} = g\vec{k}$  the expression for  $P$  can be rewritten as  $P_i = a_{ij}^* E_{0j}^{inc} + ig e_{ijk} k_j E_{0k}^{inc}$ . Using (5) one may obtain:

$$g_{xx} = \frac{1}{2V} \int_L [T_{yz,x}^{(1)}(s) - T_{zy,x}^{(1)}(s)] ds \quad (6)$$

It is worth noticing that  $T_{ik,m}^{(1)} = \int_L A_{ik} x_m ds = l_i J(s|\vec{k} = \vec{n}_m, \vec{E} = ix_m \vec{n}_k / \omega)$ , where  $\{\vec{n}_i\}$  is a basis of the coordinate system. Thus to calculate  $g$  one needs solving (2) for the following cases:

$$\begin{aligned} \vec{k} = \{k, 0, 0\}; \vec{E} = \{0, ix/\omega, 0\} & \quad \vec{k} = \{0, k, 0\}; \vec{E} = \{0, 0, iy/\omega\} \\ \vec{k} = \{0, 0, k\}; \vec{E} = \{iz/\omega, 0, 0\} & \quad \vec{k} = \{k, 0, 0\}; \vec{E} = \{0, 0, ix/\omega\} \\ \vec{k} = \{0, k, 0\}; \vec{E} = \{iy/\omega, 0, 0\} & \quad \vec{k} = \{0, 0, k\}; \vec{E} = \{0, iz/\omega, 0\} \end{aligned}$$

The results of the calculations are presented in Figure 1.

To evaluate the magnetic moment  $M$  we present the current  $j_i(s) = J(s)l_i(s)$  as a sum of an average part  $\langle j_i \rangle$  and a fluctuated one:  $\vec{j} = \langle \vec{j} \rangle + \delta\vec{j}$ . The first part  $\langle j_i \rangle$  determines the symmetrical part of  $P$ . It is reasonable to connect the

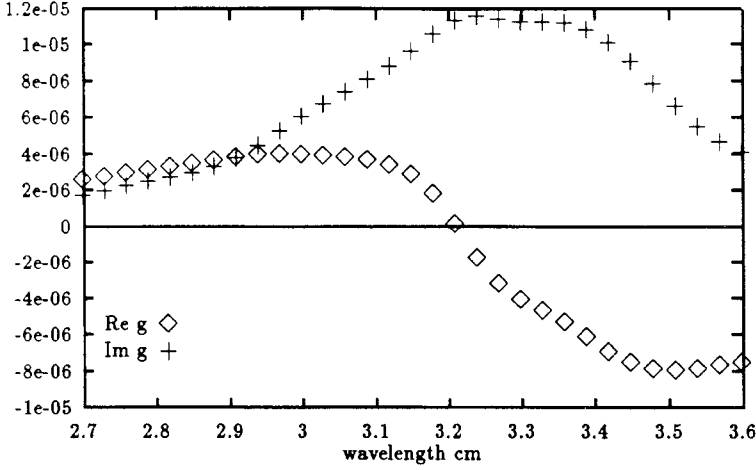


Figure 1: Chiral factor for the helix with radius  $r_h = 0.03$  cm, the length of the wire  $\mathcal{L} = 1.5$  cm, the helix step  $h = 0.03$  cm.

fluctuating part with magnetic moment  $M$  in a standard manner  $\delta \vec{j} = c \text{curl} \vec{M}$ . The consequence of the relationship is a reversal one  $\vec{M} = \frac{\mu}{2cV} \int_L [\vec{r}(s) \times \delta \vec{j}]$ . Opposite to the consideration of [7] so defined quantity is independent of the position of the coordinate origin. Introducing  $\vec{x}(s) = \vec{r}(s) - \langle \vec{r} \rangle$ , where  $\langle \vec{r} \rangle$  is the position of the center of the helix, it is possible to write

$$\begin{aligned} M_i &= \frac{\mu}{2cV} e_{ikl} \int_L r_k \delta j_l ds = \frac{\mu}{2cV} e_{ikl} \int_L (\langle r_k \rangle + x_k)(j_l - \langle j_l \rangle) ds = \\ &= \frac{\mu}{2cV} e_{ikl} \left[ \int_L x_k j_l ds + \langle r_k \rangle \int_L j_l ds - \langle r_k \rangle \langle j_l \rangle - \langle j_l \rangle \int_L x_k ds \right] = \\ &= \frac{\mu}{2cV} e_{ikl} \int_L x_k j_l ds \end{aligned}$$

where  $j$  is the whole current and  $x$  is a position of the point  $s$  in accordance with the center of the helix. To confine to the first order in  $(ak)$  it is sufficient to take into account  $E^{(0)inc}$  only. Using (2) we can rewrite the expression for  $M$

$$M_i = \frac{i\omega}{2cV} e_{ikl} \int_L \int_L x_k A_{ij}(s, s') E_{0j} ds ds' = \frac{k}{2} (e_{ilk} \alpha_{ij,k}^* E_{0j} + e_{ilk} \alpha_{ij,k}^a E_{0j})$$

The first term contributes to permeability and describes the magnetic moment appearing due to eddy currents, the second one is responsible for chirality and may be rewritten as  $0.5(gE_{0i} - g_{ij}E_{0j})$ . For bi-isotropic system we have  $\vec{M} = igk\vec{E}_0 = -ig[\vec{k} \times [\vec{n} \times \vec{E}_0]] = -i[\vec{\beta} \times \vec{H}_0]$ .

Thus the antisymmetrical parts of  $P$  and  $M$  are described by the same coupling constant  $g$ . It is well to bear in mind that we have restricted ourselves by microscopic consideration and that electric  $E$  and magnetic  $H$  fields are the microscopic fields need averaging.

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**BACKSCATTERING OF THE THIN WIRE HELIX : ANALYTICAL MODEL,  
NUMERICAL STUDY AND FREE SPACE MEASUREMENTS.  
APPLICATION TO CHIRAL COMPOSITE MODELLING.**

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**I - Introduction**

In the last 10 years, there has been an increasing interest in electromagnetic chirality. In particular, the interaction of an isotropic chiral material with electromagnetic waves. The chiral media can have various shapes, as slabs, cylinders or spheres [1-3]. In previous studies [4-6], it has been shown that a chiral inclusion can be expressed in terms of electric and magnetic dipole moments. The electromagnetic properties of the inclusions (in those cases helices) was calculated in the quasi-stationary approximation, when the microstructure size is small as compared to the wavelength. In this paper we study the electromagnetic properties of the thin wire helix without restrictions of its size compared to the wavelength : an original Integral Equation Method is used to solve analytically the problem, numerical calculations are also performed.

**II - Integral Equation Method : an Analytical Model**

Our purpose was to calculate the current as induced by an EM wave illuminating a thin wire helix perfectly conducting (figure 1) [7-8]. In order to calculate the current we start with the integral equation as provided by :

$$\mathbf{E}_{\text{tot}}^t(\mathbf{r} = \mathbf{r}_s) = \mathbf{E}_{\text{inc}}^t(\mathbf{r} = \mathbf{r}_s) - j\omega \mathbf{A} - j \frac{1}{\omega\mu\epsilon} \nabla (\nabla \cdot \mathbf{A}) \quad (1)$$

where  $\mathbf{E}_{\text{tot}}^t(\mathbf{r} = \mathbf{r}_s)$  is the tangential component of the total electric field at the surface of the helix,  $\mathbf{E}_{\text{inc}}^t(\mathbf{r} = \mathbf{r}_s)$  the tangential component of the incident electric field and  $\mathbf{A}$  the potential vector given by

$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \int_S \frac{\exp(-j\beta R)}{R} \mathbf{J}_s(\mathbf{r}') d\mathbf{r}' \quad (2)$$

where  $\mathbf{J}_s(\mathbf{r}')$  is the current density induced at the surface of the helix,  $\beta = \omega\sqrt{\epsilon\mu}$  and  $R = |\mathbf{r} - \mathbf{r}'|$ .

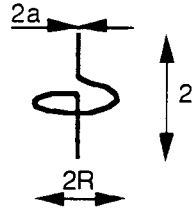


figure 1 : the thin wire helix.

The wire is thin and consequently we are tempted to make the filament approximation : the currents  $\mathbf{J}_s(\mathbf{r}')$  are concentrated on the axis of the filament and we take the boundary conditions  $\mathbf{E}_{\text{tot}}^{\perp}(\mathbf{r} = \mathbf{r}_s) = 0$  at the surface of the helix whose radius is "a". So we have three integral equations to solve, one for each wire (linear part) and one on the loop. Using cylindrical coordinates  $(\mathbf{e}_\rho, \mathbf{e}_\theta, \mathbf{e}_z)$  for currents on the linear portions, we show [7] that the current densities  $J_\theta$  et  $J_\rho$  on the straight portions are negligible as compared to  $J_z$ . For the circular loop, which indeed is a torus of radius "R" and cross-section  $\pi a^2$ , we define axis tangential to the torus  $(\mathbf{e}_\rho, \mathbf{e}_\theta, \mathbf{e}_\phi)$ , we also show that the current densities  $J_{\theta 1}$  et  $J_{\rho 1}$  on the loop are negligible as compared to  $J_\phi$ . Using similar methods developed by Hallen and Einarsson [9] we solve the integral equations on the linear parts. For the circular loop, we expand the integral equation in fonction of  $a/R$  ( $\ll 1$ ). By imposing the boundary conditions, zero value for the currents at each end of linear portions and current continuity between wires and loop, we obtain the value of the current along the helix.

By integrating the currents on the thin wire helix, the scattering field in the far zone is calculated. The equivalent electric and magnetic dipole moments  $\mathbf{p}$  et  $\mathbf{m}$  are also provided by the current distribution on the wire helix: in the low frequency approximation we provide new simple analytical expressions for  $\mathbf{p}$  et  $\mathbf{m}$ . Representative results are presented.

### III - Numerical Calculations

Two numerical codes are used in this study. The first one, named "Ficelle", solve numerically the integral equation at the surface of the helix with the thin wire approximation. The second one "Arlene" is a surface integral equation code.

Current distributions and backscattering of several chiral objects are presented for different polarizations and incidences of the incident EM wave. Results from different numerical codes are compared and discussed.

#### **IV - Comparaison between Modelling and Measurements**

Free space measurements are processed on a thin wire helix in CESTA anechoic chamber. The results are provided in the frequency range of 1.7 GHz - 20 GHz. Generally speaking, the agreement between the theoretical and measured values is quite good. An example is presented in figure 2, however a light difference occurs at resonance. In this talk, the comparison of results is discussed and physical insights into these results are provided.

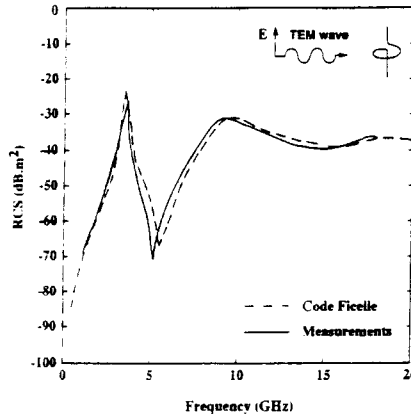


figure 2 : theoretical and measured backscattering of a thin wire helix.

#### **V - Modeling chiral composites**

In order to satisfy the concept of effective medium, it has been supposed the helix size was small compared to the wavelength in the material. In this case, an isotropic lossy chiral composite consisting of chiral objects can be represented by the following constitutive relations

$$\mathbf{D} = \epsilon_c \mathbf{E} + i\xi_c \mathbf{B} \quad \text{and} \quad \mathbf{H} = i\xi_c \mathbf{E} + \mathbf{B}/\mu_c \quad (3)$$

where  $\epsilon_c = \epsilon_0(\epsilon' + i\epsilon'')$ ,  $\mu_c = \mu_0(\mu' + i\mu'')$  and  $\xi_c = \xi'_c + i\xi''_c$  represent complex permittivity and permeability, and chirality admittance. In this talk, we give the relationships between  $\epsilon_c$ ,  $\mu_c$ ,  $\xi_c$  and the dimensions of the helix.

For low frequency, the analytical model (see II) provide analytical expressions for  $\mathbf{p}$  and  $\mathbf{m}$ . By average over helices orientation angles, we finally find  $\mathbf{P}$  and  $\mathbf{M}$  for a collection of  $N$  non-interacting helices randomly oriented in an host medium. So we can define  $\epsilon_c$ ,  $\mu_c$ ,  $\xi_c$  of the effective chiral medium. Modelling and measurements of a chiral composite are presented.

## **VI - Conclusions**

In this talk, we present an overview of Microwave Chirality Research at CEA-CESTA : First, modelling of heterogeneous chiral materials by analytical and numerical calculations of electromagnetic scattering of a chiral element (thin helix), secondly free space measurements of chiral scatterer and third the modelling of chiral composites.

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## Artificial magnetics based on circular film elements

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Artificial spatial structures in an electromagnetic field, whose wavelength exceeds substantially the dimensions of the elements and distances between them, manifest properties characteristic of dielectric and magnetic media. Such structural media are created and studied with the aim of obtaining electromagnetic materials possessing new properties of which natural dielectrics and magnetics are devoid.

An artificial magnetic structural medium based on united four cubic lattices of ring - shaped conducting elements are studied theoretically in [1, 2]. The conducting elements are considered of nonmagnetic metals. The magnetic properties of that kind medium have diamagnetic character, and its magnetic losses are great, but are not resonant, that was confirmed by experiments [3]. The sufficiently complete theory of this artificial magnetic medium and additional experimental results are given in [4]. The theory based on stationary circular currents, forming magnetic dipole moments, and on the phenomenological Lorentz - Lorenz formula.

In the present paper some additional results on the medium of circular nonresonant currents [4] and some results on a new medium of broken circular resonant currents are given. Unlike the first medium, which can be as diamagnetic only, the second medium can be as diamagnetic, so paramagnetic. The paramagnetism is a result of the resonant dependence of a current in the broken ring - shaped elements. If an equivalent electric circuit of the element of the first medium contains an inductance and a resistance only, an equivalent electric circuit of the element of the second medium contains a capacity in addition to the inductance and resistance. The capacitor is formed by parallel surfaces in the broken part of the ring - shaped elements.

If axes of the circular elements are parallel, the medium is anisotropic and it has diagonal tensor of permeability with components  $\mu_x = \mu_y = 1$ ,  $\mu_z = \mu = \mu' - i\mu''$  where according to [4, 5]

$$\mu = \frac{\chi}{1 - \chi/3}, \quad (1)$$

$$\chi = -\gamma(i\omega L) \begin{cases} 1/(R + i\omega L) & \text{for 1 - st medium,} \\ 1/[R + i(\omega L - 1/\omega C)] & \text{for 2 - d medium,} \end{cases} \quad (2)$$

$\omega$  is the angular frequency;  $R = 2\pi b/(h\sigma)$  is the ring-shaped film strip resistance ( $h \ll d \ll b$ ,  $b$  is the ring radius),  $\sigma$  is the specific conductivity;  $L = L_0 + M$ ,  $L_0 = \mu^0 b (\ln 8b/d - 1/2)$  and  $M = \mu^0 b f(2b/l)/4\pi$  are the internal and mutual inductances of elements, the function  $f(2b/l)$  is tabulated in [4, 6],  $l$  is the lattice period;  $C = \epsilon^0 S/h_0$  is the capacity  $\gamma = \pi^2 b^3 N/q$ ,  $N$  - is the elements concentration (a number of elements in a cubic meter),  $q = L/\mu^0 b$ ;  $\mu^0$  and  $\epsilon^0$  are the vacuum parameters,  $0 \leq \gamma \leq 1$ ,  $\gamma \approx 1$  is corresponding to the concentration limit.

The function  $\mu''(\omega)$  has a maximum for  $\omega = \omega_\mu$ , where

$$\omega_\mu = \alpha \begin{cases} R/L & \text{for 1 - st medium,} \\ \omega_0 [(\beta^2 + 3)^{1/2} - \beta]^{1/2} & \text{for 2 - d medium,} \end{cases} \quad (3)$$

$$\alpha = \left[1 + \frac{\gamma}{3}\right]^{-1/2}, \quad \beta = 1 - \frac{1}{2}\left[\frac{\alpha}{Q}\right]^2, \quad (4)$$

$$\omega_0 = \frac{1}{(LC)^{1/2}}, \quad Q = \frac{1}{R}\left[\frac{L}{C}\right]^2.$$

Unlike of the first medium [1, 2, 4] for the second medium the function  $\mu'(\omega)$  has a maximum for  $\omega = \omega_{max}$ , where

$$\omega_{max} = \omega_0 \alpha (1 + \alpha/Q)^{-1/2}, \quad (5)$$

and, if  $\alpha/Q < 1$ , a minimum for  $\omega = \omega_{min}$ , where

$$\omega_{min} = \omega_0 \alpha (1 - \alpha/Q)^{-1/2}. \quad (6)$$

It is interesting to consider some asymptotical cases.

1. If  $Q \rightarrow 0$ , we have from (3) - (6)

$$\omega_{\mu} \rightarrow (1 + \gamma/3)^{-1} R/L, \omega_{max} \rightarrow 0, \mu'_{max} \rightarrow 1. \quad (7)$$

Thus the medium is diamagnetic.

2. If  $Q \rightarrow \infty$ , we have from (3) - (6)

$$\omega_{\mu} \rightarrow \omega_0 (1 + \gamma/3)^{-1/2}, \omega_{min} \rightarrow \omega_{max} \rightarrow \omega_{\mu}. \quad (8)$$

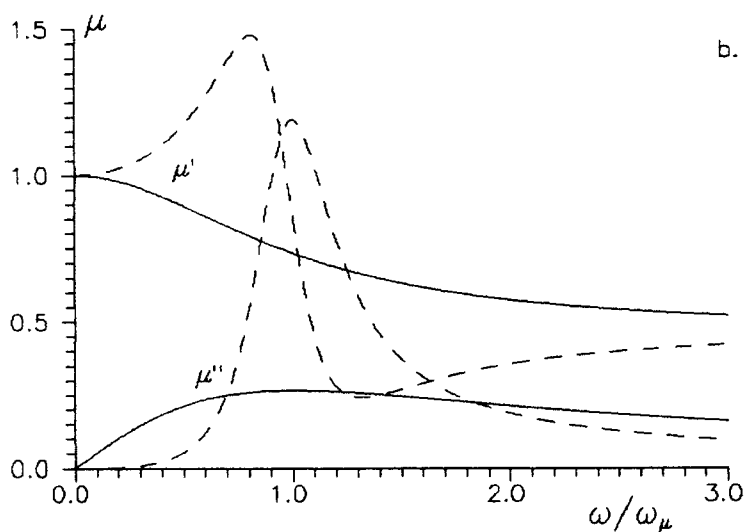
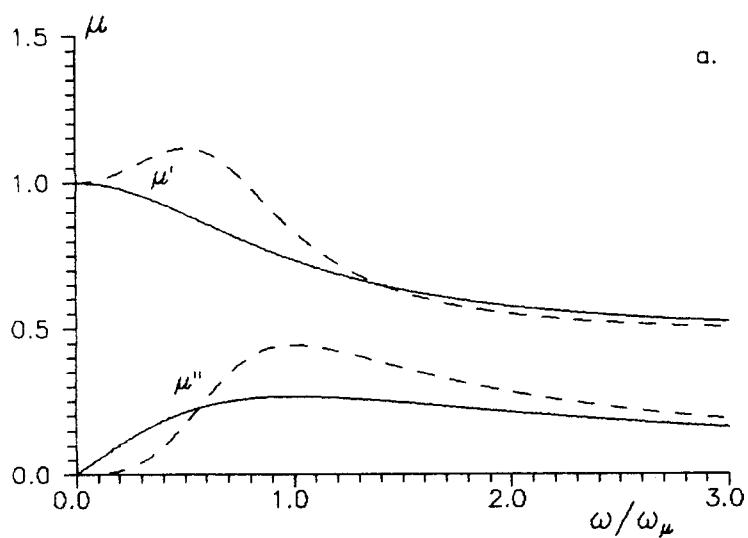
Thus the medium has properties, which are typical for media with an anomalous dispersion [7].

3. If  $Q \sim 1$ , the medium has properties as diamagnetic, so paramagnetic.

In conclusion it should be noted, that the use of Lorentz - Lorenz formula is valid, if  $|\chi| \leq 1$  [8].

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Calculated frequency dependencies of the real and imaginary parts of the permeability: solid lines are for  $\gamma = 0.65$ ,  $Q = 0$ , dashed lines are for  $\gamma = 0.65$ ,  $Q = 2/3$  (a),  $\gamma = 0.65$ ,  $Q = 2$  (b).

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Abnormally high electromagnetic activity in chiral  
media inactive at low frequency

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1. A new chiral composite material with directed multigone spirals has been proposed, manufactured and experimentally investigated in microwave diapason.

2. The dielectric permittivity and magnetic microwave permeability of the medium mentioned above have been measured by the resonator method.

3. Peculiarities of the dielectric permittivity and magnetic permeability behaviour depending on the swirl angle have been studied in centimeter range.

For measurements based on the resonator method and on the with perturbation theory the samples of the mentioned above medium were manufactured in form of hollow cylinder with lateral walls from directed isolated multigone spiral twists ( 1,5 turns ) placed between two dielectric films (see Figure 1). The cylinder diameter was 2.3 mm, length  $l$  was of order of 20 - 80 mm, the wire diameter was 0.015 mm. The spiral length  $l = a/\sin\alpha$ , spiral pitch ( $a \cot\alpha$ ), where  $a=8$  mm.  $\alpha$  is the spiral swirl angle. The turns concentration was of 40 turns per centimeter and corresponded to the metal volume fraction of 0.2 %.

The samples under investigation all had the same diameter and differed from each other by the value of  $\alpha$  and by the spiral pitch and length, consequently.

The half - wave rectangular resonators and panoramic standing wave ratio (SWR) for the 3 - 5 GHz frequency range were used [1,2]. The electric and magnetic microwave fields orientations were either perpendicular or parallel to the cylinder axis  $z$ . In Figure 2, a typical measured dependence of the reflection coefficient  $R$  on the frequency  $f$  is presented. It demonstrates the effect scale. All of reported below dependen-

ces of  $\epsilon$  and  $\mu$  components were obtained by processing the curves analogous to the Figure 2 curve. So, the dependences of the real and imaginary parts of dielectric permittivity  $\epsilon'_1, \epsilon''_1$  (for field  $E$  perpendicular to the  $z$  - axis) and magnetic permeability  $\mu'_1, \mu''_1$  ( for field  $H$  parallel to the  $z$ - axis ) on  $\alpha$  are depicted in Figure 3 for the frequency 4.4 GHz. In Figure 3 we see the following.

i) There are the  $\mu'_1$  and  $\mu''_1$  dependences on  $\alpha$ . While  $\alpha$  is small enough ( from 0 up to 20 degrees ) and the spiral turns are strongly stretched the differences of  $\mu'_1$  from 1 and  $\mu''_1$  from 0 are practically not observed.

ii) When  $\alpha$  increases  $\mu'_1$  and  $\mu''_1$  increase too. The maximal  $\mu'_1$  and  $\mu''_1$  magnitudes are reached in the range  $\alpha = 50^\circ..70^\circ$  where  $\mu''_1$  is of order of  $\mu'_1$ . The paramagnetic effect ( $\mu'_1 - 1 \gg 0$ ) and magnetic losses ( $\mu''_1 \gg 0$ ) are observed. In our former work the artificial diamagnetic based on the closed circular currents was realized. It displayed the magnetic losses but no differences of  $\mu$  from 1 were not revealed [3].

iii) The weak dependences of  $\epsilon'_1$  and  $\epsilon''_1$  on  $\alpha$  are observed. - When  $\alpha$  increases ( the spiral step decreases )  $\epsilon'$  and  $\epsilon''$  increases are small.

iiii) There is an area of values of  $\alpha$  (50 ... 70 degrees) where high magnitudes of magnetic permeability  $\mu'_1$  closed to dielectric permittivity  $\epsilon'_1$  are reached. Let us note that in typical composites based on iron small balls dielectric permittivity  $\epsilon'$  exceeds the magnetic permeability  $\mu'$  in 15 ... 20 times, and  $\mu$  reaches magnitudes of the same order if the metal volume fraction is of 60%.

In Figure 4 the dependences of  $\mu'_1, \mu''_1$  ( a, b ) and  $\epsilon'_1, \epsilon''_1$  on the frequency for the samples of different  $\alpha$  are plotted. These curves testify the weak dependency of the results described on the frequency and the possibility of the Figure 3 results extrapolating to other frequencies.

In Figure 5 we see the dependences of  $\epsilon'_1$  and  $\epsilon''_1$  on  $\alpha$  for the frequency 4.4 GHz and the field  $E$  parallel to the  $z$ -axis. In this situation these dependences are rather strong. While  $\alpha$  changes from 0 to  $90^\circ$  and spiral turn changes from strongly stretched one to ring the value  $\epsilon'_1$  falls from 120 to 1.5 and  $\epsilon''_1$  falls from 7 to  $3 \cdot 10^{-2}$ . These results were

repeated when other frequencies were used.

We see from Figures 4 c and 5 that when  $\alpha$  is close to  $90^\circ$  (the spiral pitch is small) we have practically isotropic case  $\epsilon'_\perp = \epsilon'_\parallel$ . When  $\alpha = 0, 20^\circ$ , there is large dielectric anisotropy:  $\epsilon'_\parallel \gg \epsilon'_\perp, \epsilon''_\parallel \gg \epsilon''_\perp$ . While  $\alpha$  increases the gradual transition from anisotropic case to isotropic one takes place.

Thus, in our opinion the most essential results of the present work are the following.

1. The new composite material which is a chiral medium with directed multigone spiral turns is investigated. Such composites were neither experimentally nor theoretically investigated before.

11. First the microwave paramagnetic effect (difference of the magnetic permeability  $\mu'$  from 1,  $\mu' - 1 \gg 0$ ) is discovered by direct measurements in artificial composites with out magnetic components.

111. The gigantic magnetic losses induced by microwave field in chiral composites were discovered. As was shown, these losses exceed that in traditional composites of the magnetic type having the same metal volume fraction more than in two orders. In composites based on the iron balls with volume concentration of 0.2%  $\mu' = 1$  and  $\mu'' < 0.005$

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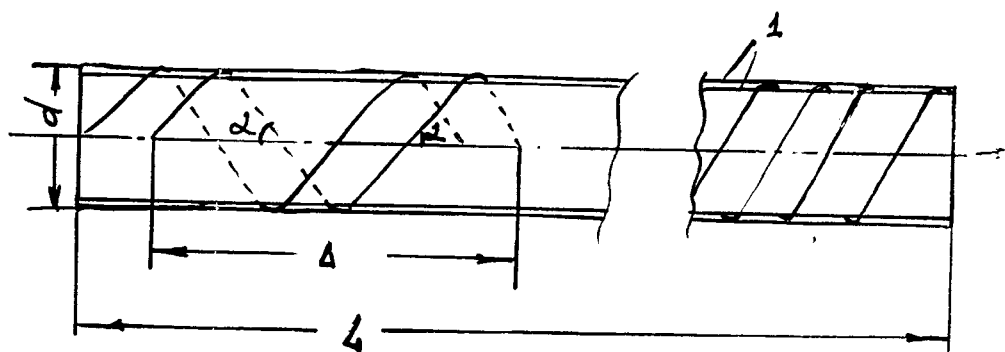


Fig. 1

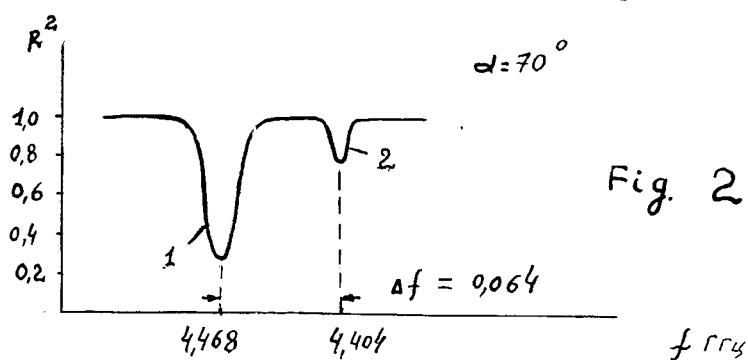


Fig. 2

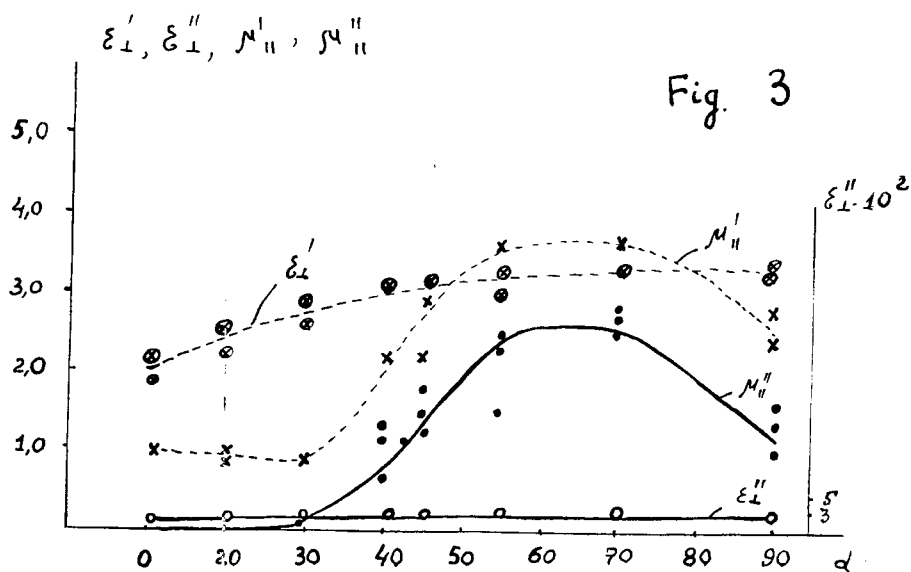


Fig. 3



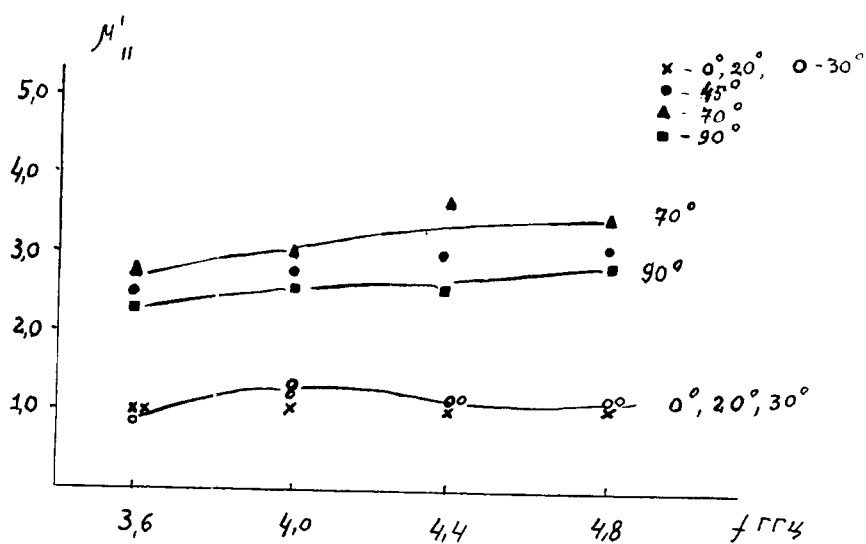


Fig. 4a

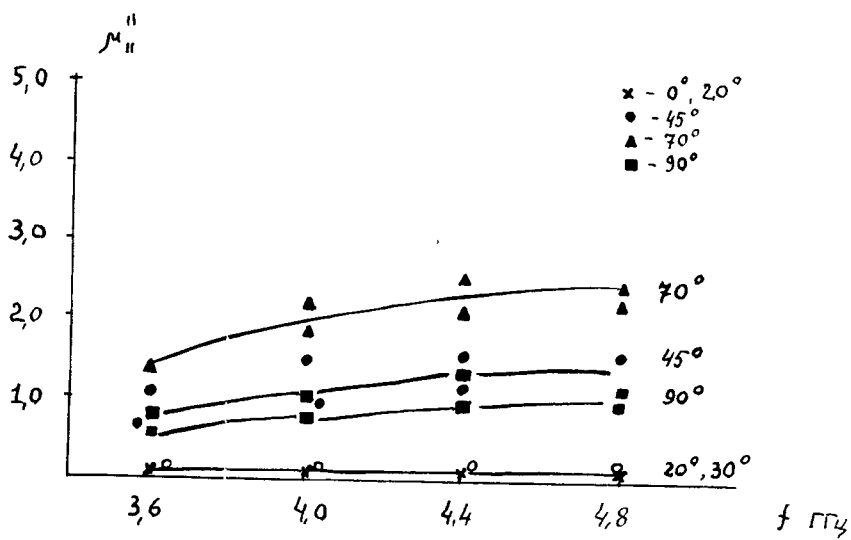


Fig. 4b

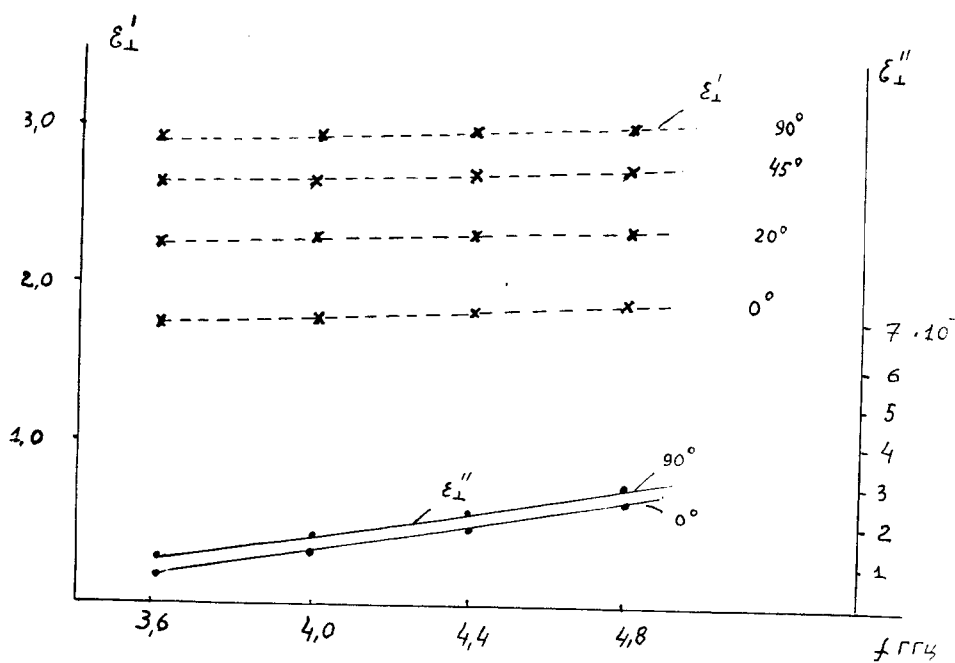


Fig. 4 c

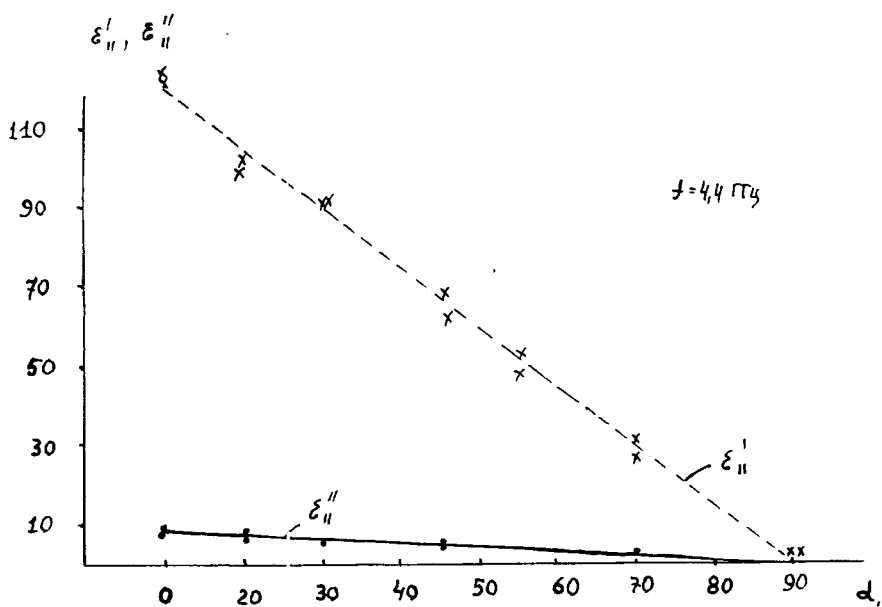


Fig. 5

# Optimal Polarisations of a General Biisotropic Half-Space

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**ABSTRACT** - The optimal polarisations of a general biisotropic half-space are calculated. The concept of optimal polarisation base is introduced to enable the optimal polarisations to be calculated from the eigenvalues of the power scattering matrix.

## 1 Introduction

In studying the effects of general biisotropic (BI) media on the polarisation characteristics of an incident wave, the classical approach is to start from a given polarisation description of the incident wave together with given constitutive parameters of the BI medium. The reflected or transmitted wave is then analysed. In polarimetric radar applications, RCS, as well as in certain problems of remote sensing, the inverse problem is often of greater importance: given a medium with arbitrary medium parameters, find the incident wave polarisation(s) that give(s) rise to certain *received* polarisation and/or power characteristics of interest. This is basically an estimation problem. The solutions for the incident waves that give rise to certain special cases of received wave polarisations are usually referred to as optimal polarisations [1].

Assuming a zero surface current density at the single interface, the reflection matrix for LP plane waves normally incident is defined from [2]:-

$$\begin{pmatrix} E_{||}^r \\ E_x^r \end{pmatrix} = \begin{pmatrix} r_{11}^l & r_{12}^l \\ r_{21}^l & r_{22}^l \end{pmatrix} \begin{pmatrix} E_{||}^i \\ E_x^i \end{pmatrix} \quad (1)$$

with

$$r_{11}^l = r_{22}^l = \frac{Z_L Z_R - Z_1^2}{(Z_L + Z_1)(Z_R + Z_1)} \quad (2)$$

$$r_{12}^l = -r_{21}^l = j \frac{Z_1(Z_R - Z_L)}{(Z_L + Z_1)(Z_R + Z_1)} \quad (3)$$

where  $Z_1$  represents the wave impedance of the medium of incidence. This result has been verified independently. It follows that co-polarisation nulls are found for  $Z_1 = \sqrt{Z_L Z_R}$ , whereas cross-polarisation nulls occur only if  $Z_R = Z_L$ . It is noted that 'co' and 'cross' refer to the directions parallel and perpendicular to the polarisation direction of an LP incident wave, respectively.

## 2 Optimal Polarisations for a BI Half-Space

### 2.1 Optimal Polarisations

For any given scattering geometry for a given scatterer, a set of polarisations of the incident wave, called *optimal polarisations* [1], exists for which:-

- the received power is maximum (*max polarisations*);
- the received power is zero (*co-pol null polarisations*);
- the reflected wave shows no depolarisation (*cross-pol null polarisations*).

The received power is defined as that obtained from the *algebraic* sum of the reflected fields (voltages) measured at both orthogonal output terminals of the receiving two-channel polarimetric antenna.

### 2.2 Optimal Polarisation Base

Strictly, the optimal polarisations can only be calculated for a complex symmetric reflection matrix [3]. To make the theory applicable to reciprocal and nonreciprocal BI media, we choose the polarisation base to be *optimal*, ie such that the associated reflection matrix is complex symmetric. It follows from analysis that, for example, a CP polarisation base is a suitable optimal polarisation base  $B^o$ , for which the CP reflected field components  $E_R^r$ ,  $E_L^r$  and incident field components  $E_R^i$ ,  $E_L^i$  are related as:-

$$\begin{pmatrix} E_R^r \\ E_L^r \end{pmatrix} = \underline{\underline{R}}^o \begin{pmatrix} E_R^i \\ E_L^i \end{pmatrix} \quad (4)$$

with

$$\underline{\underline{R}}^o = \begin{pmatrix} r_{11}^o & r_{12}^o \\ r_{21}^o & r_{22}^o \end{pmatrix} = \begin{pmatrix} r_{11}^l - jr_{12}^l & 0 \\ 0 & r_{11}^l + jr_{12}^l \end{pmatrix} \quad (5)$$

being the transformation of the reflection matrix in Eqn (1) to the CP optimal polarisation base. At normal incidence we have, with the aid of Eqns (2-3):-

$$r_{11}^o = \frac{(Z_L Z_R - Z_1^2) + Z_1 (Z_R - Z_L)}{(Z_R + Z_1)(Z_L + Z_1)} \quad (6)$$

$$r_{22}^o = \frac{(Z_L Z_R - Z_1^2) - Z_1 (Z_R - Z_L)}{(Z_R + Z_1)(Z_L + Z_1)} \quad (7)$$

### 2.3 Polarisations for Maximum Received Power

The polarisations for maximum received power are the eigenvectors associated with the greatest of the eigenvalues  $\lambda_i^2$  of the the power scattering matrix  $(\underline{\underline{R}}^o)^\dagger \underline{\underline{R}}^o$ , where  $(\underline{\underline{R}}^o)^\dagger$  denotes the hermitian adjoint of  $\underline{\underline{R}}^o$ .

The two eigenvalues are found as  $\lambda_i^2 = |r_{ii}^o|^2$  ( $i = 1, 2$ ), hence the polarisation for maximum received power is one of the CP eigenpolarisations of the BI medium:-

$$\underline{u}_1^l = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -j \end{pmatrix}, \quad \text{or} \quad \underline{u}_2^l = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ j \end{pmatrix} \quad (8)$$

It can be shown that the other CP eigenmode gives rise to a *local* extremum for the received power only and that the minimum received power is zero.

### 2.4 Polarisations for Zero Co-Polarisation

The co-pol nulls are formally defined as the polarisation vector solutions  $\underline{v}^o$  of  $\underline{\underline{R}}^o \underline{v}^o = \alpha (\underline{v}^o)^\star_\perp$ , where  $(\underline{v}^o)^\star_\perp$  denotes the vector which is hermitian orthogonal to  $\underline{v}^o$  and  $\alpha$  is a scalar to be determined. After substituting Eqn (5), the normalised solutions are found as:-

$$\underline{v}_1^l = c_1 \begin{pmatrix} j \frac{M+1}{M-1} \\ 1 \end{pmatrix}; \quad \underline{v}_2^l = c_2 \begin{pmatrix} j \frac{M-1}{M+1} \\ 1 \end{pmatrix} \quad (9)$$

with

$$M = \sqrt{\frac{Z_1(Z_R - Z_L) - (Z_L Z_R - Z_1^2)}{Z_1(Z_R - Z_L) + (Z_L Z_R - Z_1^2)}} \quad (10)$$

$$c_1 = \frac{\exp[-j\psi]}{\sqrt{\left|\frac{M+1}{M-1}\right|^2 + 1}}; \quad c_2 = \frac{\exp[-j\psi]}{\sqrt{\left|\frac{M-1}{M+1}\right|^2 + 1}} \quad (11)$$

In the special case of a reciprocal BI, *ie* a unimpedant ( $Z_R = Z_L$ ), half-space the co-pol nulls are characterised by  $M = \pm 1$ , hence the co-pol nulls are two hermitian orthogonal, linear polarisations.

## 2.5 Polarisations for Zero Cross-Polarisation

The cross-polarisation nulls are determined with the aid of the eigenvectors (polarisation vectors) for maximum received power as:-

$$\underline{w}_1^o = \underline{U}^{-1} \underline{u}_1^o \quad ; \quad \underline{w}_2^o = \underline{U}^{-1} \underline{u}_2^o \quad (12)$$

with

$$\underline{U} = \left( \underline{u}_1^o \mid \underline{u}_2^o \right) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (13)$$

Hence, for a BI half-space, these coincide with the optimal polarisations for maximum received power, *ie* the RCP and LCP eigenpolarisations of the BI medium:-

$$\underline{w}_1^l = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -j \end{pmatrix} ; \quad \underline{w}_2^l = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ j \end{pmatrix} \quad (14)$$

## 3 Conclusion

The optimal polarisations of a general biisotropic halfspace for use in monostatic polarimetric radar have been determined. It was proven that both CP polarisations give rise to zero cross-polarisation and that one of these gives rise to maximum received power. The polarisations for zero co-pol polarisation are in general EP. Their co-polarised components in a LP polarisation base are found to be mutually reciprocal-opposite.

## 4 Acknowledgement

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# Plane electromagnetic waves in uniaxial bianisotropic media

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*Electromagnetic waves in novel materials which have properties of chiral composites and omega structures are considered. The materials can be modelled by uniaxial bianisotropic constitutive relations. Both chiral and omega composites possess features promising for applications, and one may expect still more interesting effects in more general media.*

## INTRODUCTION

Electromagnetics of complex media attracts a lot of attention and efforts of researches. Among those novel microwave materials, isotropic chiral composites were intensively studied in the last decade. They proved to be useful in microwave technology, antenna design and, especially, as prospective materials for anti-reflection coverings. A novel concept of omega media with  $\Omega$ -shaped metal elements embedded in a dielectric matrix was recently introduced in [1]. These bianisotropic materials can find novel applications in microwave engineering [1, 2]. A uniaxial modification of omega composites was proposed in [3]. The uniaxial symmetry and additional interaction between orthogonal electric and magnetic fields make the materials potentially useful for non-reflecting coverings and antenna radomes [3]. Some other special cases of uniaxial bianisotropic media were considered in [4]-[7].

Here we study electromagnetic waves in the most general reciprocal uniaxial materials. In practice, such media with arbitrary linear magneto-electric interaction can be realized as microstructures with both types of inclusions — helices and  $\Omega$ -particles [6]. Another way is to arrange short metal helices in a certain order instead of choosing a random distribution. Such a composite is chiral, but the field coupling due to chirality is effective only for the fields in the  $(x - y)$  plane. Corresponding material equations become uniaxial with the most general uniaxial dyadic coupling terms:

$$\begin{aligned}\overline{D} &= \overline{\epsilon} \cdot \overline{E} + j\sqrt{\epsilon_0\mu_0}(-\kappa_t\overline{I}_t - \kappa_n\overline{z}_0\overline{z}_0 + K\overline{J}) \cdot \overline{H}, \\ \overline{B} &= \overline{\mu} \cdot \overline{H} + j\sqrt{\epsilon_0\mu_0}(\kappa_t\overline{I}_t + \kappa_n\overline{z}_0\overline{z}_0 + K\overline{J}) \cdot \overline{E}.\end{aligned}\quad (1)$$

In reciprocal media, the dielectric permittivity  $\overline{\epsilon}$  and the magnetic permeability  $\overline{\mu}$  are symmetric uniaxial dyadics

$$\overline{\epsilon} = \epsilon_0(\epsilon_t\overline{I}_t + \epsilon_n\overline{z}_0\overline{z}_0), \quad \overline{\mu} = \mu_0(\mu_t\overline{I}_t + \mu_n\overline{z}_0\overline{z}_0), \quad (2)$$

Here,  $\bar{z}_0$  stands for the unit vector along the geometrical axis,  $\bar{I}_t = \bar{x}_0\bar{x}_0 + \bar{y}_0\bar{y}_0$  is the transverse unit dyadic, and  $\bar{J} = \bar{z}_0 \times \bar{I}_t = \bar{y}_0\bar{x}_0 - \bar{x}_0\bar{y}_0$  is the 90 degree rotator in the  $(x - y)$  plane.

### PLANE WAVE SOLUTIONS

Because of the uniaxial symmetry it appears natural to split the fields into normal and transverse parts with respect to the axis  $z$ :  $\bar{E} = E_n\bar{z}_0 + \bar{E}_t$ ,  $\bar{H} = H_n\bar{z}_0 + \bar{H}_t$ . To study plane waves, we Fourier transform the Maxwell equations in the transverse plane  $(x - y)$  and eliminate the longitudinal field components. This approach is most convenient in view of potential applications of the uniaxial materials, when layered structures are formed so that the axis is normal to the interfaces.

Substituting a plane wave solution in the form  $\exp(-j\beta z)$  and solving the eigenvalue equation leads to the following expression for the normal component of the propagation factor:

$$\frac{\beta_{1,2}^2}{k_0^2} = \epsilon_t\mu_t + \kappa_t^2 - K^2 + \left( \kappa_t\kappa_n - \frac{1}{2}(\epsilon_t\mu_n + \epsilon_n\mu_t) \right) \frac{k_t^2}{nk_0^2} \pm \sqrt{D}, \quad (3)$$

where

$$D = \frac{k_t^4}{4n^2k_0^4}(\epsilon_t\mu_n - \epsilon_n\mu_t)^2 + \kappa_t^2\epsilon_n\mu_n\frac{k_t^4}{n^2k_0^4} - \kappa_t\frac{k_t^2}{nk_0^2} \left( 2\kappa_t + \kappa_n\frac{k_t^2}{nk_0^2} \right) (\epsilon_t\mu_n + \epsilon_n\mu_t) - 4K^2\kappa_t \left( \kappa_t + \kappa_n\frac{k_t^2}{nk_0^2} \right) + \epsilon_t\mu_t \left( 2\kappa_t + \kappa_n\frac{k_t^2}{nk_0^2} \right)^2,$$

$n = \epsilon_n\mu_n - \kappa_n^2$  and  $\bar{k}_t$  is the transverse propagation factor. The last formula is in agreement with the corresponding equations for some special cases known from the literature [3]-[7]. It has been also checked against a solution obtained by a different approach by Viitanen and Koivisto<sup>1</sup>.

As is seen, the analysis of general uniaxial media is rather involved. In non-chiral omega media, eigenwaves are linearly polarized *TM*- and *TE*-waves [3], whereas in isotropic chiral media eigenwaves are circularly polarized. In uniaxial chiral omega structures eigenwaves, in general, have elliptical polarization patterns.

### VECTOR TRANSMISSION-LINE PARAMETERS. REFLECTION AND TRANSMISSION

Next we study plane-wave reflection and transmission phenomena in planar uniaxial bianisotropic layers. Such problems for complex media layers can be effectively treated using the vector transmission-line theory, introduced for biisotropic materials in [8]. In that theory, a plane layer of composite material is modelled by an equivalent transmission line with dyadic wave impedances and dyadic propagation factors. In general, the modelling line is non-symmetric, i.e. its wave impedances depend on the direction of the wave propagation. For non-reciprocal media, the same is true for the propagation dyadics as well. In the present case, the medium

<sup>1</sup>Private communication, October 1993



is reciprocal, and the general plane-wave solution to the Maxwell equations can be written as a sum of two waves travelling in the opposite directions of the  $z$ -axis:

$$\vec{E}_t = e^{-j\vec{\beta}z} \cdot \vec{A} + e^{j\vec{\beta}z} \cdot \vec{B}, \quad (4)$$

where the two-dimensional constant vectors  $\vec{A}$  and  $\vec{B}$  are determined by the boundary conditions. The exponent functions of the two-dimensional dyadic propagation factors  $\vec{\beta}$  and  $\vec{\beta}$  are understood in terms of their Taylor expansions.

Dyadic wave impedances (or admittances) determine relations between the transverse electric and magnetic field components in plane waves propagating in unbounded medium. For an eigenwave we can write

$$\vec{E}_t = \mp \vec{Z}_{\pm} \cdot \vec{z}_0 \times \vec{H}_t \quad - \vec{z}_0 \times \vec{H}_t = \pm \vec{Y}_{\pm} \cdot \vec{E}_t \quad \vec{Y}_{\pm} = \vec{Z}_{\pm}^{-1} \quad (5)$$

where the upper and the lower signs correspond to the waves propagating in the positive and negative directions of the axis  $z$ , respectively. Dyadic impedances and admittances can be found from the Fourier transformed Maxwell equations after substitution the propagation factors (3).

In general, the eigenwaves are different for the waves travelling in the opposite directions of the  $z$ -axis. We therefore are forced to introduce notations like  $\vec{E}_{1t}$  and  $\vec{E}_{2t}$  for the transverse eigenfields propagating in the positive direction, and  $\vec{E}_{1t}$  and  $\vec{E}_{2t}$  for the reflected waves. Since the two eigenwaves possess different propagation factors  $\beta_1$  and  $\beta_2$ , the propagator should act at a sum of two eigenfields  $\vec{E}_{1t}$  and  $\vec{E}_{2t}$  as

$$e^{-j\vec{\beta}z} \cdot (\vec{E}_{1t} + \vec{E}_{2t}) = e^{-j\beta_1 z} \vec{E}_{1t} + e^{-j\beta_2 z} \vec{E}_{2t} \quad (6)$$

For the opposite propagation direction,

$$e^{j\vec{\beta}z} \cdot (\vec{E}_{1t} + \vec{E}_{2t}) = e^{j\beta_1 z} \vec{E}_{1t} + e^{j\beta_2 z} \vec{E}_{2t} \quad (7)$$

The propagator dyadic in (6) can be explicitly written as the sum of two dyads:

$$e^{-j\vec{\beta}z} = e^{-j\beta_1 z} \vec{E}_{1t} \vec{E}_{1t}' + e^{-j\beta_2 z} \vec{E}_{2t} \vec{E}_{2t}' \quad (8)$$

and

$$e^{j\vec{\beta}z} = e^{j\beta_1 z} \vec{E}_{1t} \vec{E}_{1t}' + e^{j\beta_2 z} \vec{E}_{2t} \vec{E}_{2t}' \quad (9)$$

where the two-dimensional vectors  $\vec{E}_{1t}'$ ,  $\vec{E}_{2t}'$  form a basis reciprocal to the basis  $(\vec{E}_{1t}, \vec{E}_{2t})$ .

With the determined equivalent parameters of the vector transmission line, the reflection and transmission dyadics can be expressed in a way similar to that known from the conventional transmission-line theory, but with dyadic parameters. For details we refer to [8]. Some numerical examples demonstrate characteristic features of plane waves in uniaxial bianisotropic materials.

## CONCLUSION

In this paper, the theory of electromagnetic wave propagation in the most general reciprocal uniaxial medium was constructed. Using the vector transmission-line theory, reflection and transmission coefficients for plane layers were analysed. The novel composites are expected to have rather interesting properties, since their material equations are still more general than that of isotropic chiral materials and uniaxial omega structures. As it is known, both special cases can offer important potential applications.

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# Energy Dissipation and Absorption in Bi-Isotropic Media

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## ABSTRACT

Several sets of constitutive relations may be used for describing the interaction of a time-harmonic electromagnetic wave with a reciprocal bi-isotropic, or chiral, medium. However, the permittivity, permeability, and chirality coefficient of such a medium carry different meanings when they are expressed using different formalisms. This has an impact on energy dissipation requirements in a passive chiral medium. The influence of the chirality parameter on the reflectivity of a Dallenbach-type chiral absorber is also relative to the chosen set of constitutive equations.

## 1. EXAMPLES OF CONSTITUTIVE RELATIONS AND CONNECTIONS BETWEEN THE DIFFERENT SETS

From an electromagnetics viewpoint, an homogeneous chiral material can be described by specific equations incorporating 3 macroscopic constitutive parameters. One possibility is [1]

Formalism 1 :

$$\begin{aligned} \mathbf{D} &= \epsilon_1 \mathbf{E} + i\gamma \mathbf{H} \\ \mathbf{B} &= -i\gamma \mathbf{E} + \mu_1 \mathbf{H} \end{aligned} \quad (1)$$

The  $\mathbf{D}$  and  $\mathbf{H}$  fields can also be expressed as a function of  $\mathbf{E}$  and  $\mathbf{B}$  in a covariant form [2]

Formalism 2 :

$$\begin{aligned} \mathbf{D} &= \epsilon_2 \mathbf{E} + i\xi \mathbf{B} \\ \mathbf{H} &= i\xi \mathbf{E} + \frac{1}{\mu_2} \mathbf{B} \end{aligned} \quad (2)$$

A third possibility, putting emphasis on the non-local character of the medium is [3]

Formalism 3 :

$$\begin{aligned} \mathbf{D} &= \epsilon_3 (\mathbf{E} + \beta \nabla \times \mathbf{E}) \\ \mathbf{B} &= \mu_3 (\mathbf{H} + \beta \nabla \times \mathbf{H}) \end{aligned} \quad (3)$$

Other expressions exist, but in order to avoid possible confusion, we will restrict the discussion to the 3 expressions mentioned above.

These 3 sets can be shown to be equivalent for time-harmonic fields and connecting equations can be derived [1]. Using an  $e^{-i\omega t}$  time dependence, one obtains with our notations

$$\begin{aligned} \epsilon_1 &= \epsilon_2 + \mu_2 \xi^2, \mu_1 = \mu_2, \gamma = \xi \mu_2 \\ \epsilon_2 &= \epsilon_1 - \frac{\gamma^2}{\mu_1}, \mu_2 = \mu_1, \xi = \frac{\gamma}{\mu_1} \end{aligned} \quad (4)$$

$$\begin{aligned}\epsilon_1 &= \frac{\epsilon_3}{1 - \epsilon_3\mu_3\beta^2\omega^2}, \mu_1 = \frac{\mu_3}{1 - \epsilon_3\mu_3\beta^2\omega^2}, \gamma = \frac{\epsilon_3\mu_3\beta\omega}{1 - \epsilon_3\mu_3\beta^2\omega^2} \\ \epsilon_3 &= \epsilon_1 \left(1 - \frac{\gamma^2}{\epsilon_1\mu_1}\right), \mu_3 = \mu_1 \left(1 - \frac{\gamma^2}{\epsilon_1\mu_1}\right), \beta = \frac{\gamma}{\omega\epsilon_1\mu_1} \frac{1}{1 - \frac{\gamma^2}{\epsilon_1\mu_1}}\end{aligned}\quad (5)$$

and

$$\begin{aligned}\epsilon_2 &= \epsilon_3, \mu_2 = \frac{\mu_3}{1 - \epsilon_3\mu_3\beta^2\omega^2}, \xi = \omega\epsilon_3\beta \\ \epsilon_3 &= \epsilon_2, \mu_3 = \frac{\mu_2}{1 + \mu_2 \frac{\xi^2}{\epsilon_2}}, \beta = \frac{\xi}{\omega\epsilon_2}\end{aligned}\quad (6)$$

As far as the chirality parameters are concerned, it is apparent from the examination of equations (4) to (6) that their meaning depends on the formalism; some of them do not solely represent the handedness of the medium, but also contain some information relative to the dielectric and/or magnetic character of the medium. The meaning of the permeability and the permittivity is also relative to the formalism in the general case. As for specific cases, it can be seen that in formalisms 2 and 3 the permittivities are equal, while in formalisms 1 and 2 the permeabilities are equal. For the particular case of low chirality, particularly important from an experimental viewpoint [4], and quantified by the relations  $\epsilon_3\mu_3\beta^2\omega^2 \ll 1$ ,  $\mu_2\xi^2/\epsilon_2 \ll 1$ , or  $\gamma^2/\epsilon_1\mu_1 \ll 1$ , it is worth noting that there is only little difference between the permittivities and permeabilities in the various formalisms.

In addition, the intrinsic wave impedance can be computed using each formalism separately, which yields  $\eta_1 = (\mu_1/\epsilon_1)^{1/2}$ ,  $\eta_2 = (\mu_2/(\epsilon_2 + \mu_2\xi^2))^{1/2}$ , and  $\eta_3 = (\mu_3/\epsilon_3)^{1/2}$ . Using the connecting equations (4) to (6), it follows that  $\eta_1 = \eta_2 = \eta_3$ , i.e. the impedances are formalism-independent. Since the wavenumbers of the two canonical right- and left-circularly polarized (LCP and RCP) waves propagating in a chiral medium,  $k_-$  and  $k_+$ , do not depend on the choice of particular constitutive equations, it is always possible to define a set of three complex scalars which are formalism-independent, i.e.  $k_-$ ,  $k_+$  and the wave impedance.

## 2. PASSIVITY REQUIREMENTS FOR CHIRAL MEDIA

Requiring that the net time-average power flux entering a closed surface  $S$  with interior volume  $V$  in a chiral medium without sources be positive, the requirement on the constitutive parameters of a passive chiral medium can be found (by passive medium, it is meant a medium that is passive for all fields). After some manipulations, one obtains for formalism 1

$$\begin{aligned}\text{Im}(\epsilon_1) &\geq 0, \quad \text{Im}(\mu_1) \geq 0 \\ \text{Im}(\epsilon_1)\text{Im}(\mu_1) &\geq (\text{Im}(\gamma))^2\end{aligned}\quad (7)$$

This set of inequalities agrees with that given by Lindell [5]. If there is no chirality ( $\gamma = 0$ ), then the results known for regular dielectric and magnetic media,  $\text{Im}(\epsilon_1) \geq 0$  and  $\text{Im}(\mu_1) \geq 0$ , are retrieved. Using the connecting equations and (7), one gets for formalisms 2 and 3

$$\begin{aligned} \text{Im}(\epsilon_2 + \mu_2 \xi^2) &\geq 0, \quad \text{Im}(\mu_2) \geq 0 \\ \text{Im}(\epsilon_2 + \mu_2 \xi^2) \text{Im}(\mu_2) &\geq (\text{Im}(\xi \mu_2))^2 \end{aligned} \quad (8)$$

and

$$\begin{aligned} \text{Im}\left(\frac{\epsilon_3}{1 - \epsilon_3 \mu_3 \beta^2 \omega^2}\right) &\geq 0, \quad \text{Im}\left(\frac{\mu_3}{1 - \epsilon_3 \mu_3 \beta^2 \omega^2}\right) \geq 0 \\ \text{Im}\left(\frac{\epsilon_3}{1 - \epsilon_3 \mu_3 \beta^2 \omega^2}\right) \text{Im}\left(\frac{\mu_3}{1 - \epsilon_3 \mu_3 \beta^2 \omega^2}\right) &\geq \left(\text{Im}\left(\frac{\epsilon_3 \mu_3 \beta \omega}{1 - \epsilon_3 \mu_3 \beta^2 \omega^2}\right)\right)^2 \end{aligned} \quad (9)$$

It should be noticed that only  $\epsilon_1$  and  $\mu_1$  have the same meaning in terms of dissipation of energy as have the permittivity and the permeability in ordinary isotropic dielectric and magnetic materials. Also, only in formalism 1 is chirality totally uncoupled from other properties. In formalism 2, only  $\mu_2$  has the same meaning as the usual magnetic permeability. In formalism 3, neither  $\epsilon_3$  nor  $\mu_3$  can be considered as regular permittivity and permeability. Therefore, as least regarding energy dissipation is concerned, one may regard formalism 1 as the best suited to the description of reciprocal biisotropic media.

### 3. NORMAL INCIDENCE REFLECTION COEFFICIENT OF CHIRAL MEDIA

In this section, we consider a metal-backed chiral slab of thickness  $e$ , on which a normally incident linearly polarized plane wave is impinging. Our goal is to check if and how chirality could be useful for applications related to microwave reflection reduction. Using the generic symbol  $\eta$  for the intrinsic wave impedance, one obtains the following expression for the reflection coefficient

$$R = \frac{\frac{\eta - \eta_0}{\eta + \eta_0} - e^{2ik_{eq}e}}{1 - \frac{\eta - \eta_0}{\eta + \eta_0} e^{2ik_{eq}e}}, \quad k_{eq} = \frac{k_- + k_+}{2} = \omega \sqrt{\epsilon_1 \mu_1} = \omega \sqrt{\mu_2(\epsilon_2 + \mu_2 \xi^2)} = \frac{\omega \sqrt{\epsilon_3 \mu_3}}{1 - \epsilon_3 \mu_3 \omega^2 \beta^2} \quad (10)$$

The reflection coefficient is written as a function of 2 formalism-independent variables: the impedance  $\eta$  and an equivalent wavenumber  $k_{eq}$ , which is the average of the 2 canonical LCP and RCP wavenumbers; its formalism-independent character clearly appears when written in this fashion.

Reminding the results of section 1 and considering the expression of  $k_{eq}$  in (10) using formalism 1, it can be seen that the chirality coefficient  $\gamma$  appears neither in the impedance, nor in the equivalent wavenumber. Extending the analysis to formalism 2, it is found that the chirality parameter  $\xi$  both appears in the expressions of the impedance and the equivalent wavenumber. As for formalism 3, because  $\eta_3$  does not depend on  $\beta$ , it can be seen that the chirality parameter  $\beta$  appears in  $R$  only through the equivalent wavenumber  $k_{eq}$ .

Thus, for formalism 2, macroscopic chirality is present in the definitions of both the impedance and the equivalent wavenumber. For formalism 3, it appears only in the expression of the equivalent wavenumber. For formalism 1, the macroscopic chirality does appear neither in the expression of the impedance, nor in that of the equivalent wavenumber, and thus is not included at all in the expression of the reflection coefficient. Moreover, formalism 1 is the one

in which the permittivity and the permeability have the same meaning regarding the dissipation of energy as they have in ordinary dielectric and magnetic materials. At first sight, such a conclusion does not augur well of the superiority of biisotropic media as microwave suppressors over more conventional materials. This comment should nevertheless be qualified by mentioning that only normal incidence on reciprocal bi-isotropic media was considered, and that the general case of oblique incidence on a nonreciprocal bi-isotropic medium is still to be studied in details, not to mention the case of bi-anisotropic media.

Another comment should also be made. Only homogeneous media were dealt with so far in this article. However, in a composite containing chiral inclusions in an otherwise nonchiral matrix, the situation is different. In particular, because of the electromagnetic coupling originating from the shape of the inclusions, the macroscopic effective permittivity and the permeability will depend on both the microscopic electric and magnetic polarizabilities, as well as on the microscopic chiral polarizability of the inclusions. Thus, microscopic chirality may play a role at a macroscopic level, and inhomogeneous chiral media may yield interesting combinations of permittivity and permeability. Besides, if properly understood and controlled, other phenomena such as scattering or multiple scattering may be used to enhance global energy absorption. In any case, it turns out increasingly clear that chiral materials should present some inhomogeneous character if they are to be considered as serious candidates for reflection reduction.

## CONCLUSION

3 formalisms applicable to homogeneous chiral media were compared for time-harmonic dependence, and the relationships between the constitutive parameters expressed using the different sets of constitutive equations were recalled. The requirements for dissipation in a passive chiral medium were derived in each formalism. The reflection coefficient dependence with respect to macroscopic chirality was found to differ from one formalism to another because of the formalism-dependent meaning of the constitutive parameters. It appears that the inhomogeneous character may be important for applications involving the absorption of microwave energy in chiral coatings.

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## HOW TO TAILOR AND ORIENT METALLIC HELICES TO GET MICROWAVE CHIRALITY

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### I - INTRODUCTION

The composite chiral media we are working on are made of metallic helices embedded randomly in a host dielectric medium. In order to maximize the chiral effects, two conditions are both to be satisfied :

- helices have to be excited by the incident wave;
- the interaction between helices and the incident wave has to induce a non negligible chirality.

We present here the optimization of the design of a single turn helix with respect to the incident wave characteristics (direction and polarization). The helix is defined by its dimensions (see figure 1,  $p$  : pitch,  $d$  : diameter and  $l$  : wirelength).

We shall call "tight helices" those with a  $p/d$  ratio smaller than 1, and "lax helices" those with  $p/d$  greater than 1. The reference helix has a  $p/d$  ratio equal to 1. (figure 2)

### II - EXCITATION EFFICIENCY

In order to analyze the efficiency of the helix, we shall consider the four following basic excitations, where  $\vec{k}_i$  is the wave vector (figure 3) :

- $\vec{k}_i$  along the helix axis ( $\parallel$ Axis);
- $\vec{k}_i$  perpendicular to the helix axis ( $\perp$ Axis).

In each case for  $\vec{k}_i$ , we have two possible directions for  $\vec{E}_i$ .

This efficiency will be evaluated by computing the backscattered field, using the AWAS code (Analysis of Wire Antennas and Scatterers), in free space ( $\epsilon_0, \mu_0$ ) and assuming perfect conductive helix.

### 1) Frequency dependance

On figures 4, we can find the backscattered field of a reference helix ( $p=d=8$  mm) plotted versus the frequency for different excitations. The first resonance appears when  $l$  (wirelength) is equal to  $\lambda/2$ . The second one ( $l = \lambda$ ) is not excited in the case of  $\perp$ Axis excitation. (In this case, the current on the helix must be symmetrical). In the case of  $\parallel$ Axis excitation, and according to the direction of the incident electric field, the second resonance may be more or less important than the first one. In the case of  $\vec{E}_i \parallel AC$  (see figure 5), the coupling occurs mostly on D and B and leads to a  $\lambda$  resonance. In the case of  $\vec{E}_i \parallel DB$ , the most important point is C and the current distribution is symmetrical.

In the investigated frequency range (0 to 20 GHz), we can observe also the third resonance ( $l = 3 \lambda/2$ ). These results show that the best condition, in terms of frequency, to get coupling between an incident wave and the reference helix is obtained for the first resonance.

### 2) Effects of $p/d$ on the excitation efficiency

On the diagrams of figure 6, we can observe the fluctuations of the backscattered field versus  $p/d$ . The frequency is tuned to the first resonance.

In the best case of  $\parallel$ Axis excitation ( $\vec{E}_i \parallel DB$ ), the maximum of the backscattered field is obtained for a value of  $p/d$  close to 1. The situation is the same in one of the  $\perp$ Axis excitations ; in the other case, the value of the backscattered field increases with  $p/d$ .

## III - CHIRALITY EFFICIENCY

In order to evaluate the chirality efficiency, we plot the ratio between the cross and co-polarization components of the backscattered electric field. (We cannot observe any circular dichroism because of lossless media). This ratio is plotted versus  $p/d$  for different excitations on figure 7. We observe that the fluctuations of the cross/co-polarization ratio differs with the excitation conditions. In the case of  $\perp$ Axis excitation, we have both cases : increase or decrease of the chiral efficiency when  $p/d$  increases.

In the case of Pasteur media, with randomly distributed helices, it is better to have medium chirality for each helix whatever the excitation, than high chirality for a specific excitation and very low for others. For that reason, we shall avoid very tight or very lax helices.

## IV - COMPROMISE BETWEEN EXCITATION AND CHIRAL EFFICIENCIES

We are going to define here a single parameter which is able to take into account both effects : excitation and chirality. The most suitable is the scalar product between the incident electric field  $\vec{E}_i$  and the backscattered magnetic field  $\vec{H}_r$ .

$$C_{EH} = \vec{E}_i \cdot \vec{H}_r$$

The crosspolar component of  $\vec{H}_r$  is colinear to  $\vec{E}_i$ , and so a high chirality will increase the value of  $C_{EH}$ . On the other hand, a high coupling efficiency will increase the backscattered fields. For easier handling, we modify  $C_{EH}$  in :

$$C_{EH} = 10^2 \|\vec{E}_i\| \cdot \|\vec{E}_r\| \cos \alpha$$



with :  $\vec{E}_r$ : backscattered electric field

$$\alpha = \frac{\pi}{2} - \text{Arctg} \left( \frac{\text{crosspolar of } \vec{E}_r}{\text{co-polar of } \vec{E}_r} \right)$$

On figure 8, we see the fluctuations of  $C_{EH}$  versus  $p/d$  in both cases of excitation ( // and  $\perp$ Axis), and it is clear that the maximum is obtained for a value of  $p/d$  close to 1. This result is independent on the direction of the incident electric field  $\vec{E}_i$ .

## VI - CONCLUSION

These results show that the reference helix gives the best compromise between chiral and excitation efficiency. This means that whatever the direction and the polarization of the incident wave, provided that the frequency corresponds to the first resonance, the interaction with the helix will provide a non negligible chirality. In this case, in a randomly distributed medium, all helices will interact with the incident wave, and the global effect will be important. All these conclusion stated on the analysis of the backscattered fields stands also for the forward scattered ones.

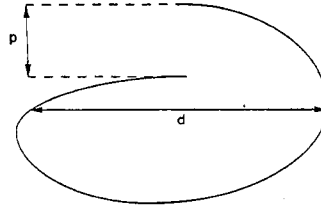


Figure 1 : Single turn helix

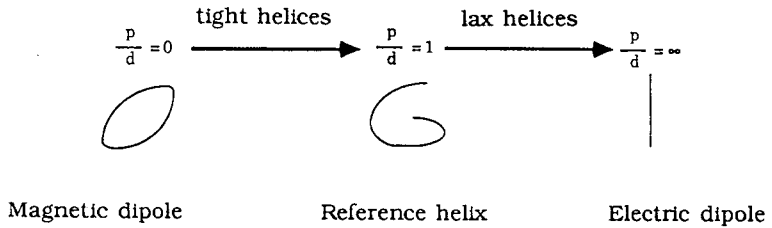


Figure 2 : Helices classification

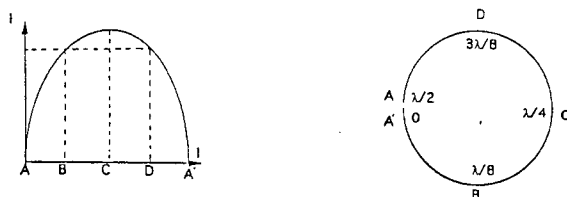


Figure 5a : Helix geometry and current distribution for even resonance

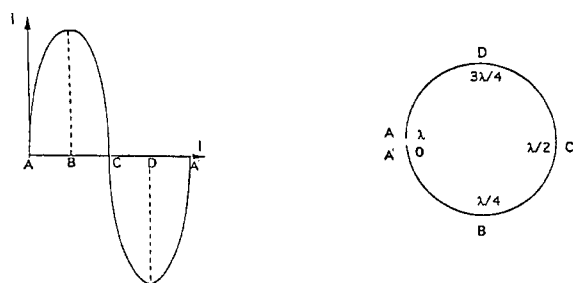


Figure 5b : Helix geometry and current distribution for odd resonance

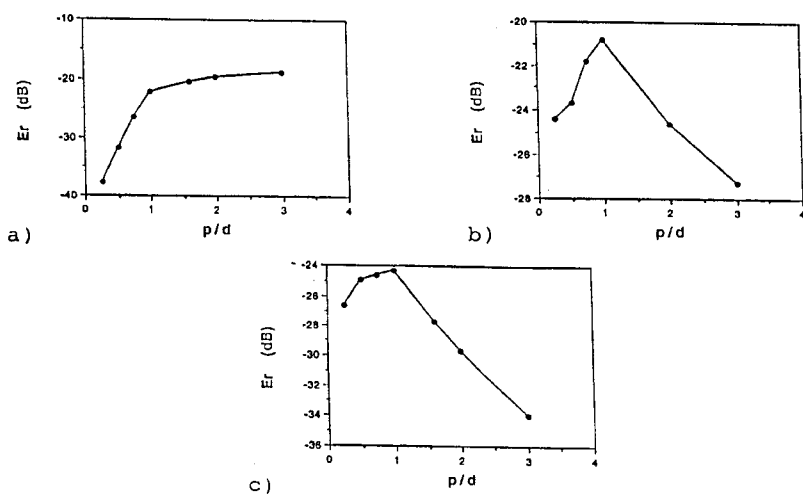


Figure 6 : Backscattered electric field at the first resonance versus  $p/d$

- a)  $\perp$  Axis excitation,  $\vec{E}_1 \parallel$  helix axis
- b)  $\perp$  Axis excitation,  $\vec{E}_1 \perp$  helix axis
- c)  $\parallel$  Axis excitation,  $\vec{E}_1 \parallel$  BD

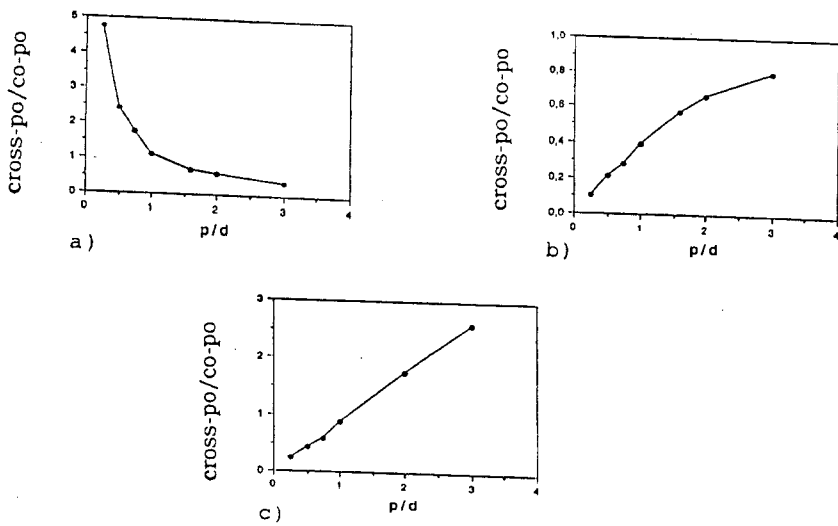


Figure 7 : Co/cross polar ratio on the backscattered electric field for the first resonance

- a)  $\perp$ Axis excitation,  $\vec{E}_1 \parallel$  helix axis
- b)  $\perp$ Axis excitation,  $\vec{E}_1 \perp$  helix axis
- c)  $\parallel$ Axis excitation,  $\vec{E}_1 \parallel$  BD

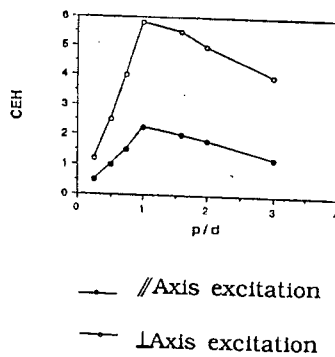


Figure 8 :  $C_{EH}$  versus  $p/d$  for the first resonance

LORENTZ-COVARIANT SOLUTION OF THE INVERSE PROBLEM OF REFLECTION  
AND TRANSMISSION FOR A DISPERSIVE BIANISOTROPIC MEDIUM

The surface impedance and characteristic matrix methods belong to a number of the most general and effective methods for solving boundary value problems in electrodynamics of stratified bianisotropic media at rest [1-5]. In Ref. [6,7] these methods are generalized on the case of uniformly moving media by making use of the exterior algebra [8]. Owing to its advantages the latter is now effectively applied in special and general relativity, but it still does not take a proper place in optics. As it proved to be the Lorentz-covariant impedance and characteristic matrix methods are especially useful in solving inverse boundary value problems for bianisotropic media [6,7]. The solutions obtained in Ref. [6] enable one, by measuring reflection and transmission coefficients, to find all 36 material parameters of a stationary bianisotropic medium provided the spatial dispersion is absent. For an uniformly moving dispersionless linear medium, the similar inverse problem is solved in Ref. [7]. In the present paper we consider the general case of dispersive bianisotropic medium with the four-dimensional constitutive equation

$$G(\underline{x}) = \int \mathcal{M}(\underline{y}) F(\underline{x} - \underline{y}) d^4 y, \quad (1)$$

where  $F$  and  $G$  are the field and induction tensors,  $\mathcal{M}$  is some tensor function of type (2,2). It is shown in Ref. [5,7,9] that dispersive anisotropic and bianisotropic media can be described by the generalized material tensors. For a field  $F$  with the

evolution operator  $\mathcal{Z}$  ( $F(\underline{x} + \underline{y}) = \mathcal{Z}(\underline{y})F(\underline{x})$ ), the generalized material tensor  $M(\mathcal{Z})$  is defined by the relations

$$G(\underline{x}) = M(\mathcal{Z})F(\underline{x}), \quad M(\mathcal{Z}) = \int \mathcal{M}(\underline{y})\mathcal{Z}(-\underline{y})d^4y. \quad (2a,b)$$

The material tensor  $M(\mathcal{Z})$  can be expressed through the reflection and transmission operators of the medium as follows.

Let a wave with the four-dimensional wave vector  $\underline{K} = \underline{\tau} + \xi \underline{Q}$  be incident onto a bianisotropic layer with the interface normal  $\underline{Q}$ . The four-dimensional impedance  $\gamma$  and the characteristic matrix  $\mathbb{C}$  [6,7] relate the boundary values of the polarization 1-forms  $\varphi = (\underline{u} \wedge \underline{v})J(Q \wedge F)$  and  $h = \underline{Q}JG$  so that

$$h = \gamma \varphi, \quad \underline{Q}\gamma = \underline{\tau}\gamma = 0, \quad \gamma \underline{Q} = \gamma \underline{\tau} = 0, \quad (3)$$

$$\begin{bmatrix} \varphi \\ h \end{bmatrix}_{\underline{x}_2} = \mathbb{C} \begin{bmatrix} \varphi \\ h \end{bmatrix}_{\underline{x}_1}, \quad (4)$$

where  $\wedge$  and  $J$  are the exterior and interior products [6-8],  $\underline{u}$  and  $\underline{v}$  are auxiliary vectors ( $\underline{u}J\underline{Q} = \underline{v}J\underline{\tau} = 1$ ,  $\underline{u}J\underline{\tau} = \underline{v}J\underline{Q} = 0$ ). The relations obtained in Refs. [6,7] enable one to express  $\gamma$  and  $\mathbb{C}$  through the reflection and transmission operators of the layer and to calculate the wave vectors  $\underline{K}_j = \underline{\tau} + \xi_j \underline{Q}$  and the polarization 1-forms  $\varphi^j$  and  $h^j$  of the partial waves excited in the layer. To find the field and induction 2-forms  $F^1$  and  $G^1$  of a partial wave, it is necessary to calculate also values  $\varphi^{1'}$  and  $h^{1'}$  of its parameters  $\varphi$  and  $h$  at some other values of  $Q$  and  $\tau$ , namely,  $Q'$  and  $\tau'$ . Then we obtain

$$F^1 = K^1 \wedge (\varphi^{1'} + f_u Q'), \quad G^1 = *[(\beta^{1'} + L_u Q') \wedge K^1], \quad (5)$$

where  $*$  is the star operator [6-8],  $f_u$ ,  $L_u$  and  $\beta^{1'}$  are some parameters depending on  $Q$ ,  $\tau$ ,  $\varphi^1$ ,  $h^1$ ,  $Q'$ ,  $\varphi^{1'}$  and  $h^{1'}$ .

Let the tensors  $F^j$  and  $G^j$  of arbitrary six eigenwaves with linearly independent amplitudes  $(F^j)$  be found. Then, superpositions of these six waves are described by the generalized material tensor

$$M(\mathbf{x}) = \sum_{j=1}^6 G^j \otimes \underline{s}_j, \quad (6)$$

where the 2-vectors  $\underline{s}_j$  are defined by the conditions  $\underline{s}_j J F^n = \delta_j^n$ . Thus, the material tensor  $M(\mathbf{x})$  is uniquely expressed, in the final analysis, through the given (measured) reflection and transmission operators of the medium. The obtained general solutions of the inverse problems can be used for developing material parameters measurement methods for both uniformly moving and motionless media.

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# Power Reflection and Absorption for Lossy Chiral Media

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**ABSTRACT** - The influence of the complex chiral admittance on the absorption capacity and absorption effectiveness of lossy chirals is investigated. Passivity bounds for the triplet  $(\epsilon, \mu, \xi)$  are presented for the Post-Jaggard formalism.

## 1 Introduction

It has repeatedly been claimed that chiral media can provide enhanced absorption performance over ordinary dielectric or magnetic media [1][2], although questions have arisen to whether this possible improvement is due to the handedness of the medium [3].

The results presented in [1][2] assumed real values of the chiral admittance  $\xi$  under time-harmonic excitation. However characterisation of  $\xi = \xi' - j\xi''$  in terms of geometrical and electromagnetic properties of chiral elements [4][5] has shown that for realistic helix-based chiral media the condition for chiral losslessness,  $\frac{\xi''}{\xi'} = 0$ , can be satisfied only under exceptional circumstances. As a further step towards the controlled design of artificial chirals, it is therefore important to consider the effect of chiral loss, *ie* a nonzero  $\xi''$ , on the wave propagation, reflection and absorption.

It is important to realize that the real and imaginary parts of  $\xi = \xi'(1 - j \tan \delta_c)$  are in general not independent parameters. Analysis of the chirality of lossy multi-turn helices [5] shows that, for example, for perfectly conducting helices in a dielectrically lossy host medium:-

$$\xi' = g \sqrt{\frac{\epsilon'_{host}}{\mu'_{host}}} \sqrt{\sqrt{1 + \tan^2 \delta_{ehost}} + 1} \quad (1)$$

$$\tan \delta_c \triangleq \frac{\xi''}{\xi'} = \frac{\sqrt{1 + \tan^2 \delta_{ehost}} - 1}{\tan \delta_{ehost}} \quad (2)$$

where  $g$  denotes a factor determined by the helix geometry. Hence for such inclusions the chiral loss tangent takes a fixed value for arbitrary  $\tan \delta_{ehost}$ . This must be born in mind when interpreting the following results.

The power absorption capacity  $A_c$  and power absorption effectiveness  $A_e$  have been defined previously for an (achiral) isotropic lossy half-space under time-harmonic excitation [6]:-

$$A_c = \frac{\text{time-averaged power loss per unit volume}}{\text{time-averaged power transmitted across the interface}} \quad (3)$$

$$A_e = \frac{\text{time-averaged power loss per volume unit}}{\text{time-averaged power incident onto the absorber}} \quad (4)$$

In particular, it was demonstrated that  $A_c$  shows to what extent a given medium may be fundamentally suitable as an EM absorber, that is independent of the environment in which this material is placed, by using a definition which shows to be very useful in practice. On the other hand,  $A_e$  was shown to exhibit finite optimal values for  $\epsilon$ ,  $\tan \delta_e$ ,  $\mu$  and  $\tan \delta_m$  at which maximum absorption performance may be achieved for arbitrary depth  $d$  and frequency  $f$  within the absorbing medium. The reflectivity  $R$  is implicitly present in the definition of  $A_e$  since  $A_e = A_c(1 - R)$ . We apply the concept of  $A_c$  and  $A_e$  to the case of a lossy chiral half-space in order to gain further insight into the effect of magneto-electric coupling on the overall loss effects.

The following constitutive matrix equation is adopted to describe the time-harmonic excitation of chiral media:-

$$\begin{pmatrix} \underline{D} \\ \underline{H} \end{pmatrix} = \begin{pmatrix} \epsilon & -j\xi \\ -j\xi & \mu^{-1} \end{pmatrix} \begin{pmatrix} \underline{E} \\ \underline{B} \end{pmatrix} \quad (5)$$

with  $\epsilon = \epsilon'(1 - j \tan \delta_e)$ ,  $\mu = \mu'(1 - j \tan \delta_m)$  and  $\xi = \xi'(1 - j \tan \delta_c)$ . Note that  $\tan \delta_c$  is a true scalar as opposed to  $\xi'$  which is a pseudo-scalar.

## 2 Power Dissipation in Lossy Chiral Media

Poynting's theorem states that  $\underline{\nabla} \cdot \underline{S} = \frac{j\omega}{2} (\underline{E} \cdot \underline{D}^* - \underline{H}^* \cdot \underline{B})$ , where  $\underline{S} = \frac{1}{2} (\underline{E} \times \underline{H}^*)$  is the complex Poynting vector, denoting the local power flux. In passive lossy media, the dissipated power  $\underline{P} = \Re[\underline{S}]$  is a positive quantity and we obtain, after separating into real and imaginary parts:-

$$\underline{\nabla} \cdot \underline{P} = \Re \left[ \frac{j\omega}{2} (\underline{E} \cdot \underline{D}^* - \underline{H}^* \cdot \underline{B}) \right] = P_e + P_m + P_{em} \quad (6)$$



where

$$P_e = \frac{\omega}{2} [\epsilon' \tan \delta_e + 2\mu' \xi'^2 \tan \delta_c + \mu' \xi'^2 \tan \delta_m (1 - \tan^2 \delta_c)] |\underline{E}|^2 \quad (7)$$

$$P_m = \frac{\omega}{2} (\mu' \tan \delta_m) |\underline{H}|^2 \quad (8)$$

$$P_{em} = \omega \mu' \xi' (\tan \delta_m + \tan \delta_c) \Im (\underline{E} \cdot \underline{H}^*) \quad (9)$$

Notice that for reciprocal biisotropic media,  $P_{em}$  does not contain a term in  $\Re (\underline{E} \cdot \underline{H}^*)$ , as opposed to its expression for general biisotropic media [7]. The possible values for  $\epsilon$ ,  $\mu$  and  $\xi$  are restricted by the conditions for passivity:  $\Im(\mu) \geq 0$ ,  $\Im(\epsilon + \mu \xi^2) \geq 0$  and  $\Im(\mu) \Im(\epsilon + \mu \xi^2) - [\Im(\mu \xi)]^2 \geq 0$ , i.e.:-

$$\mu' \tan \delta_m \geq 0 \quad (10)$$

$$\mu' \xi'^2 \tan \delta_m (1 - \tan^2 \delta_c) + 2\mu' \xi'^2 \tan \delta_c + \epsilon' \tan \delta_e \geq 0 \quad (11)$$

$$\mu'^2 \xi'^2 (1 - \tan^2 \delta_m) \tan^2 \delta_c + 4\mu'^2 \xi'^2 \tan \delta_m \tan \delta_c + \mu' \epsilon' \tan \delta_m \tan \delta_e + 2\mu'^2 \xi'^2 \tan^2 \delta_m \geq 0 \quad (12)$$

Because of the dependence of  $\xi$  on  $\epsilon$  and  $\mu$ , it is in general not possible to deduce from these conditions explicit bounds for  $\xi'$ ,  $\xi''$  or  $\tan \delta_c$  for passivity. Therefore, great care must be exercised in selecting realistic values for the triplet  $(\epsilon, \mu, \xi)$ .

Explicit but lengthy expressions have been obtained for the absorption capacity  $A_c$ , the absorption effectiveness  $A_e$  and reflectivity  $R$  by integrating over a depth  $d$  and making use of the expression for the wave impedance of chirals. The expressions for  $A_c$ ,  $A_e$  and  $R$  for isotropic media [6] have been retrieved as a special case. The analysis shows that, in general, increasing  $\xi'_r$  below unity reduces reflection from a magnetic chiral slab backed by a perfect conductor or from a magnetic chiral half-space, but not for dielectric ones (unless  $\epsilon'_r < 1$ ), in accordance with previously obtained conclusions [9]. The effect of increasing  $\tan \delta_c$  for arbitrary  $\xi'_r$  is a decrease of the reflection from either a dielectric or magnetic chiral slab.

As an illustration, Figs 1-4 show the skin depth, wavelength,  $A_c$  and  $A_e$  for dielectric chiral media for a '+' CP incident wave, whereas Figs 5-8 show the results for magnetic chirals.  $A_c$  and  $A_e$  are shown for  $d = 1$  m at  $f = 1$  GHz. The effect of changes in  $\tan \delta_c$  is seen to be comparatively small in general. Absorption essentially decreases with increasing  $\tan \delta_c$  for  $\xi'_r < \xi'_{r0}$  but increases with increasing  $\tan \delta_c$  for  $\xi'_{r0} > 1$ , where  $\xi'_{r0}$  is a constant larger than unity for dielectric chirals and smaller than unity for magnetic chirals. It is interesting to notice that the difference between dielectric chirals and magnetic chirals diminishes the more  $\tan \delta_c$  deviates from 0 for arbitrary  $\xi'_r$  and the more  $\xi'_r$  deviates from 1 for arbitrary  $\tan \delta_c$ .

### 3 Acknowledgement

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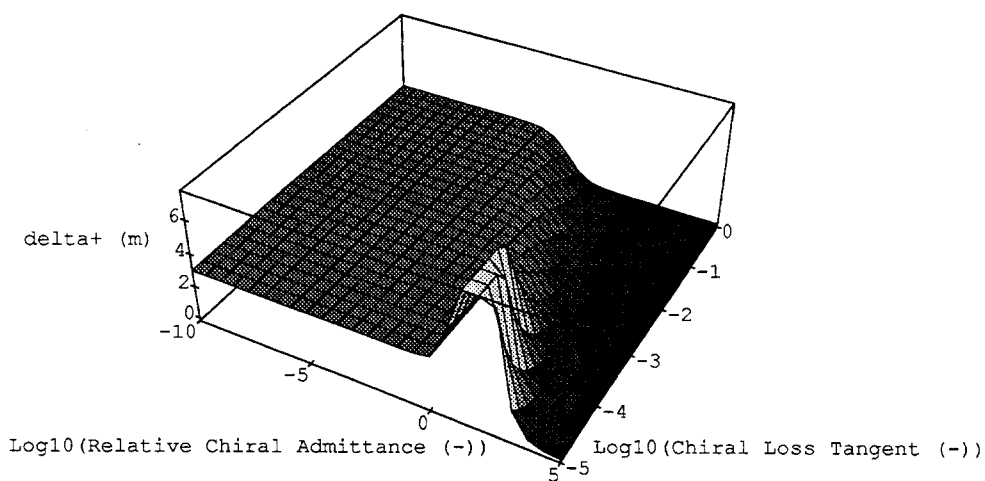


Figure 1: Skin Depth  $\delta_+$  (m) as a function of  $\xi'_r$  and  $\tan \delta_c$  for  $\epsilon = (10 - j0.1) \epsilon_o$ ,  $\mu = (1 - j0) \mu_o$ ,  $f = 1$  GHz.

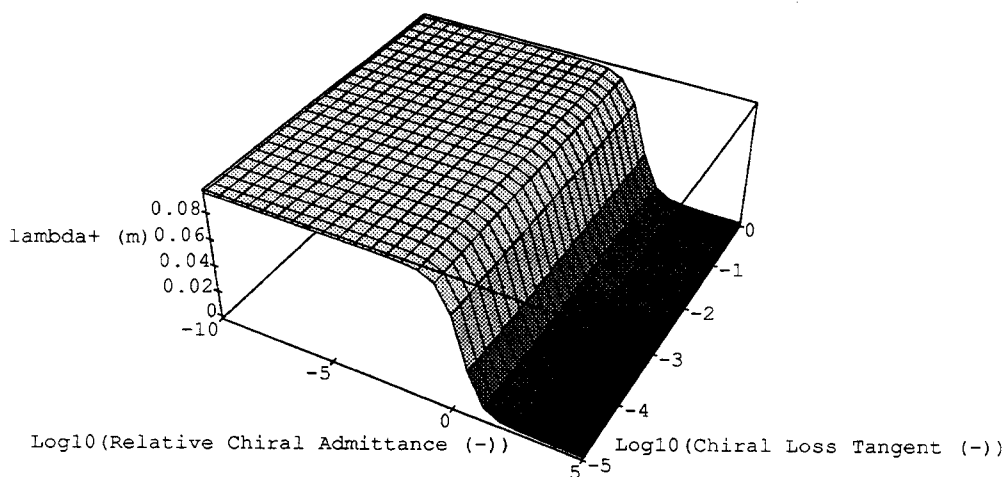


Figure 2: Wavelength  $\lambda_+$  (m) as a function of  $\xi'_r$  and  $\tan \delta_c$  for  $\epsilon = (10 - j0.1) \epsilon_o$ ,  $\mu = (1 - j0) \mu_o$ ,  $f = 1$  GHz.

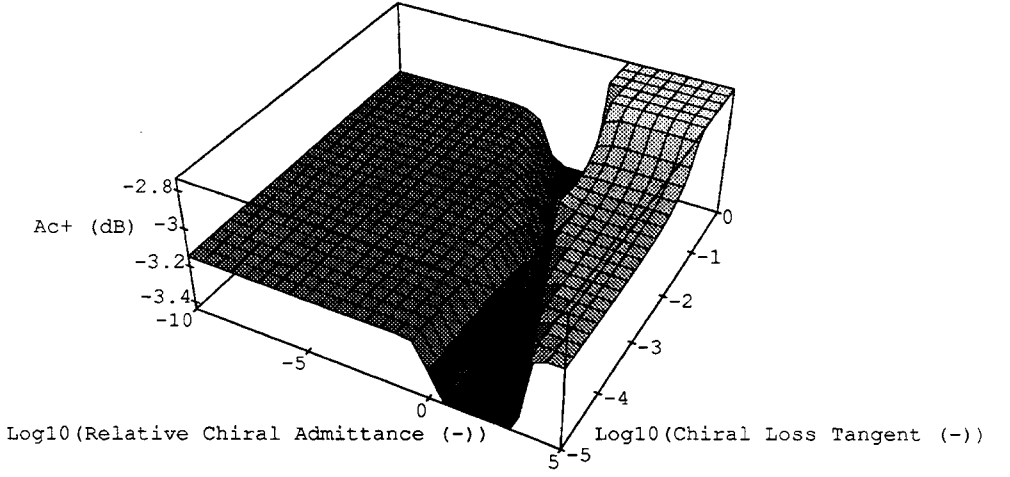


Figure 3: Absorption Capacity  $A_{c+}$  (dB/m) as a function of  $\xi'_r$  and  $\tan \delta_c$  for  $\epsilon = (10 - j0.1) \epsilon_o$ ,  $\mu = (1 - j0) \mu_o$ ,  $f = 1$  GHz.

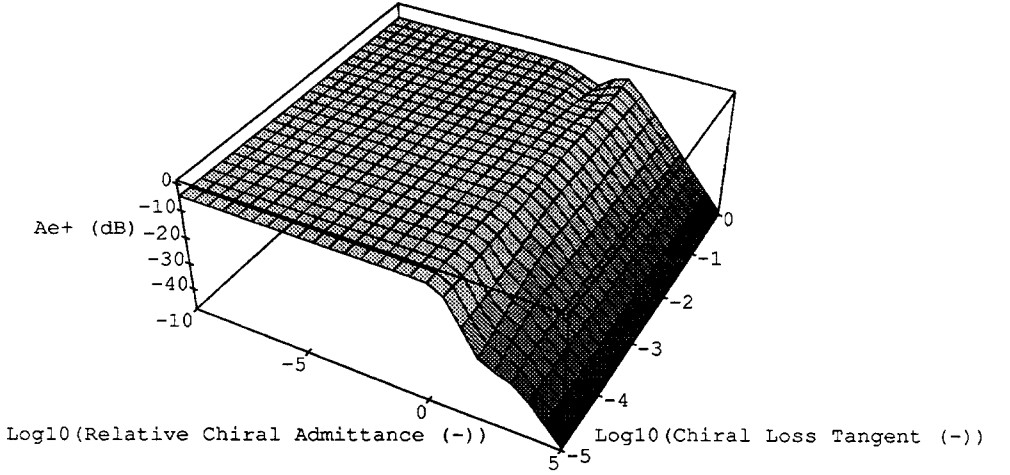


Figure 4: Absorption Effectiveness  $A_{e+}$  (dB/m) as a function of  $\xi'_r$  and  $\tan \delta_c$  for  $\epsilon = (10 - j0.1) \epsilon_o$ ,  $\mu = (1 - j0) \mu_o$ ,  $f = 1$  GHz.

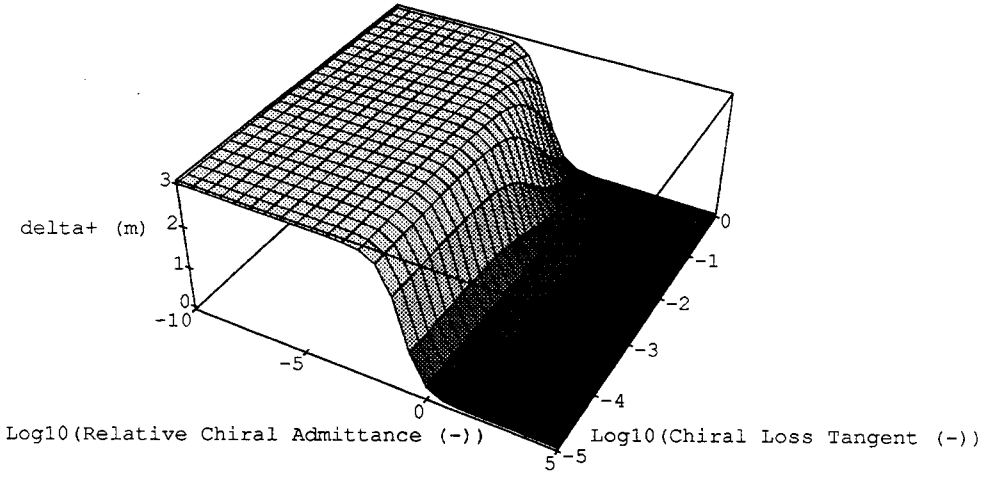


Figure 5: Skin Depth  $\delta_+$  (m) as a function of  $\xi'_r$  and  $\tan \delta_c$  for  $\epsilon = (1 - j0) \epsilon_o$ ,  $\mu = (10 - j0.1) \mu_o$ ,  $f = 1$  GHz.

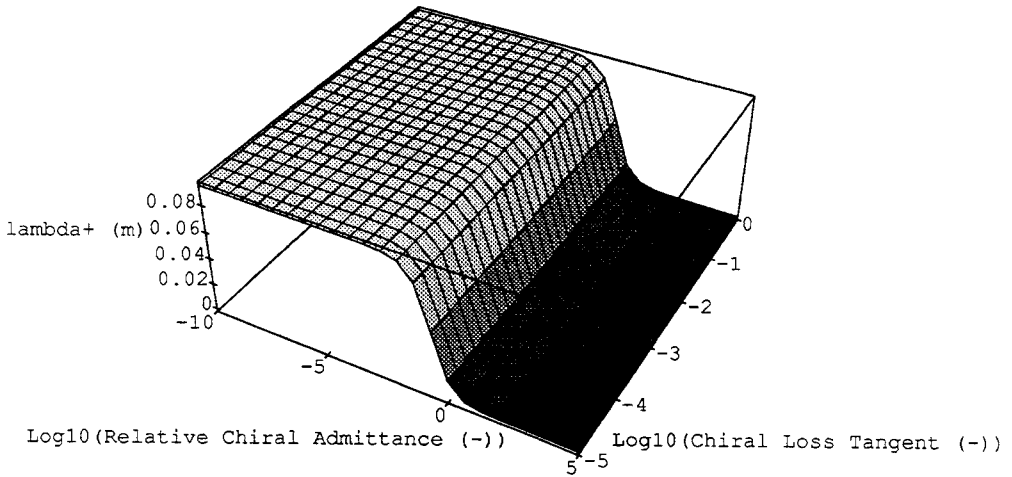


Figure 6: Wavelength  $\lambda_+$  (m) as a function of  $\xi'_r$  and  $\tan \delta_c$  for  $\epsilon = (1 - j0) \epsilon_o$ ,  $\mu = (10 - j0.1) \mu_o$ ,  $f = 1$  GHz.

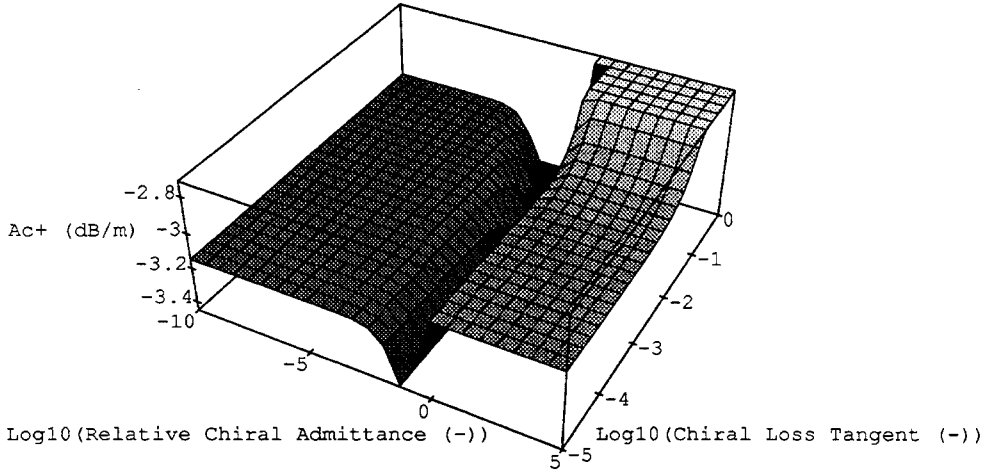


Figure 7: Absorption Capacity  $A_{c+}$  (dB/m) as a function of  $\xi'_r$  and  $\tan \delta_c$  for  $\epsilon = (1 - j0) \epsilon_o$ ,  $\mu = (10 - j0.1) \mu_o$ ,  $f = 1$  GHz.

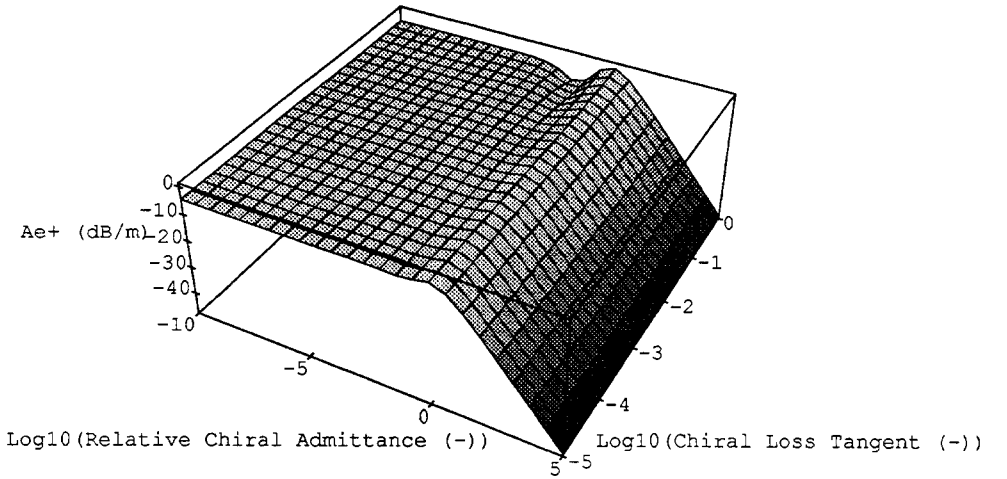


Figure 8: Absorption Effectiveness  $A_{e+}$  (dB/m) as a function of  $\xi'_r$  and  $\tan \delta_c$  for  $\epsilon = (1 - j0) \epsilon_o$ ,  $\mu = (10 - j0.1) \mu_o$ ,  $f = 1$  GHz.

# THE INTERACTION OF ELECTROMAGNETIC AND ACOUSTIC WAVES IN BIANISOTROPIC CRYSTALS WITH ELECTRICALLY-INDUCED ANISOTROPY

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The interaction of electromagnetic and acoustic waves in crystals ( acoustooptical interaction ) finds broad application for operation different kinds of materials. Now for this purpose bianisotropic crystals which have good photoelastic and gyrotropic properties are used. In the last year gyrotropic cubic crystals of silenite structure are used, such as: germanat, silicat and titanat vismut crustals and others [1,2]. If for one-axle and two-axle crystals we must take optical gyrotropy into account only for light beams directions near the optical axis in crystals , but as for cubic crystals it must be taken into account for any geometry of the acoustooptical ( hereinafter "AO" ) interaction [3].

The peculiarities of AO interaction in one-axle gyrotropic crystals under the approximation of given polarization ( elliptical ) have been considered in [4]. Using numerical decision of the differential equation system by means of electronic calculation machine ( ECM ) different geometries of AO interactions in gyrotropic cubic crystals with electrically-induced anisotropy have been investigated [5].

In this paper by using slowly changing amplitude method, Bragg diffraction in gyrotropic cubic crystals with electrically-induced anisotropy is carried out. Analitical expressions, which give the possi-

bility for calculations of energetic and polarization characteristics of diffracted waves in regime of strong and weak interactions are obtained.

Now let us see non-collinear geometry of light waves interaction in the nearness of OZ axis of the cubic crystal with acoustic wave which spread along the OX axis ( plane of diffraction is perpendicular to OY axis ). Let's assume that ultrasonic waves with displacement vector  $\vec{U} = \vec{U}_0 \exp[i(Kx - \Omega t)]$  (  $K = \Omega/v$ ,  $\Omega$  is the frequency and  $v$  is the phase velocity of ultrasound ) is in the space between the planes  $z=0$  and  $z=1$ . The ultrasonic wave makes periodic in time and space changes of dielectrical tensor permittivities  $\Delta\epsilon_{ik}$ , which is connected with the elastic deformations

$$U_{ik} = (1/2)(\nabla_k U_i + \nabla_i U_k)$$

and photoelastic constant  $p_{ijkl}$  by means of expression:

$$\Delta\epsilon_{ik} = -\epsilon_{il}\epsilon_{jk}p_{ljmn}U_{mn},$$

where  $\epsilon_{ik}$  is the dielectrical permittivities tensor.

In the region of ultrasound and light beams crossing under the influence of the electrical field  $\vec{E}^0$  on the cubic crystal the induced polarisation of medium appears

$$P_i = (-\epsilon^2/8\pi)(\beta_{ik}E_k + \beta_{ik}^*E_k^*), \quad (1)$$

where tensor  $\beta_{ik}$  is connected with  $p_{iklm}$  and electrooptical constant  $r_{ikl}$  by means of

$$\beta_{ik} = p_{iklm}U_{lm} + r_{ikl}E_l^0. \quad (2)$$



For calculations of the diffracted light amplitude we will rely upon the Maxwell equation systems

$$\text{rot} \vec{E} = -\left(1/c\right) \frac{\partial \vec{B}}{\partial t}, \quad \text{rot} \vec{H} = \left(1/c\right) \frac{\partial \vec{D}}{\partial t} \quad (3)$$

and material equations [1,7] for bianisotropic dielectric medium

$$\vec{D} = \epsilon \vec{E} + i\gamma \vec{H} + 4\pi \vec{P}, \quad \vec{B} = \vec{H} - i\gamma \vec{E}, \quad (4)$$

where  $\gamma$  is the gyrotropic parameter.

From expressions (1) and (2) wave equation for light field strain in the region of ultrasound beam follows [5]

$$\text{rot rot} \vec{E} - 2i\gamma \left(1/c\right) \frac{\partial \text{rot} \vec{E}}{\partial t} + (\epsilon^2/c^2 - \gamma^2/c^2) \frac{\partial^2 \vec{E}}{\partial t^2} = -(4\pi/c^2) \frac{\partial^2 \vec{P}}{\partial t^2}. \quad (5)$$

The decision of wave equation (5) may be written in the form [2,3,5]:

$$\vec{E} = \vec{E}_0(z) \exp[i(\vec{k}_0 \vec{r} - \omega t)] + \vec{E}_1(z) \exp[i(\vec{k}_1 \vec{r} - \omega t)], \quad (6)$$

where

$$\vec{E}_0 = A_0(z) \vec{e}_0 + B_0(z) \vec{e}_2, \quad \vec{E}_1 = A_1(z) \vec{e}_1 + B_1(z) \vec{e}_2, \quad (7)$$

and  $\vec{e}_0 = [\vec{k}_0 \vec{e}_0] / |[\vec{k}_0 \vec{e}_0]|$ ,  $\vec{e}_1 = [\vec{k}_1 \vec{e}_2] / |[\vec{k}_1 \vec{e}_2]|$ ;  $\vec{e}_2$  is the single vector, which is ortogonal to plane of AO interaction.

By introducing expression (2) into wave equation (1) the differential equation system for complex vector amplitude may be written in the form [3,8,10]:

$$\frac{d\vec{E}_0}{dz} = P\vec{E}_0 + Q\vec{E}_1,$$

$$\frac{d\vec{E}_1}{dz} = F\vec{E}_1 + L\vec{E}_0,$$

(3)

where

$$P = \begin{pmatrix} \Delta_{00} & \Delta_{02} + \rho \\ \Delta_{20} - \rho & \Delta_{22} \end{pmatrix},$$

$$Q = \begin{pmatrix} x_{01} & x_{02} \\ x_{21} & x_{22} \end{pmatrix},$$

$$L = \begin{pmatrix} x_{10} & x_{12} \\ x_{20} & x_{22} \end{pmatrix},$$

$$F = \begin{pmatrix} \Delta_{11} + \Delta & \Delta_{12} + \rho \\ \Delta_{21} - \rho & \Delta_{22} + \Delta \end{pmatrix},$$

and  $\Delta_{\mu\nu} = (i\pi/\lambda_0 n \cos\varphi_0) (\vec{e}_\mu \Delta \epsilon_e \vec{e}_\nu)$ ,  $x_{\mu\nu} = (i\pi n^3/2\lambda_0 \cos\varphi_0) (\vec{e}_\mu \Delta \epsilon_e \vec{e}_\nu)$ ,  $\Delta = (2cn \cos\varphi_0/\omega) \delta k$  ( $\delta k$  is the detuning parameter),  $\Delta \epsilon_e = -\epsilon^2 r: \vec{E}^0$  is the dielectric permittivities perturbation tensor by electric field  $\vec{E}^0$ ;  $\mu, \nu = 0, 1, 2$ . In (3) the matrixes  $P$  and  $F$  describe the anisotropic and gyrotropic crystals properties in the external electrical field  $\vec{E}^0$ , but  $Q$  and  $D$  are photoelastic properties.

Using matrix method [9-10] and boundary conditions  $\vec{E}_0(z=0) = (A_{||}, A_{\perp})^T$ ,  $\vec{E}_1(z=0) = (0, 0)^T$  ( $A_{||}, A_{\perp}$  - complex amplitudes of the incident light wave on the boundary of the AO interaction region  $z=0$ , " $\tau$ " is the symbol of the transposition operation) the decision of equation system (3) may be written in the form:

$$\begin{pmatrix} \vec{E}_0(z) \\ \vec{E}_1(z) \end{pmatrix} = \exp \begin{pmatrix} P & Q \\ D & F \end{pmatrix} z \begin{pmatrix} \vec{E}_0(0) \\ \vec{E}_1(0) \end{pmatrix}. \quad (9)$$

For 2x2- matrix  $M=F+DPD^{-1}$  with elements  $M_{11}=M_{22}$ ,  $M_{12}=M_{21}=0$ , the solution of equation system (8) may be written in the form [3]:

$$\vec{E}_0 = D^{-1} (-F \exp(-0.5Mz) (1/\sqrt{T}) \sin(z\sqrt{T}) + \exp(-0.5Mz) D^{-1} \cos(z\sqrt{T})) D \vec{E}_0(0), \quad (10)$$

$$\vec{E}_1 = \exp(-0.5Mz) (1/\sqrt{T}) (\sin(z\sqrt{T})) D \vec{E}_0(0),$$

where  $T=(DPD^{-1}F-DQ-M^2/4)$ . The matrix functions in expression (9) and (10) may be calculated from Keli-Gamilton theorem [10,11].

The forth-order system (8) may be solved exactly only in regime of small coupling ( $|x_{ij}|l \ll 1$ , where  $l$  is the length of AO interaction). Using the boundary condition  $\vec{E}_1(0)=0$ , we obtain directly [8]

$$\vec{E}_0(z) = \vec{E}_0(0) \exp(zP), \quad (11)$$

$$\vec{E}_1(z) = \exp(zF) \int_0^z \exp(z'F) D \exp(z'P) \vec{E}_0(0) dz'.$$

It should be noticed that expressions (11) may be used for calculations of many AO devices [12]. Now we demonstrate the case of AO interaction in gyrotropic cubic crystals in the (100) plane for shear acoustic waves which spread along the axis [001] and are polarized along [010]. Under this conditions external electric field  $\vec{E}^0$

must be applied along [010] direction. Such a geometry of AO interaction may be of great interest for making AO devices. Using (4) the complex-vector amplitude of diffracted wave on the boundary of AO interaction  $z=1$  may be written in the form:

$$\begin{aligned} \vec{E}_1 = & \exp(-i\delta/2) \{ [-(N\sin a_1 + \tilde{N}\sin a_2)A_1 + R(\cos a_1 - \cos a_2)A_1 - \\ & - (A\sin a_1 + \tilde{A}\sin a_2)A_1] + iA_1(B\sin a_1 + \tilde{B}\sin a_2) \} \vec{e}_1 + \\ & + \exp(-i\delta/2) \{ [(N\sin a_1 + \tilde{N}\sin a_2)A_1 + R(\cos a_1 - \cos a_2)A_1 + \\ & + (A\sin a_1 + \tilde{A}\sin a_2)A_1] + i(B\sin a_1 + \tilde{B}\sin a_2)A_1 \} \vec{e}_2, \end{aligned}$$

where

$$\begin{aligned} A = 2\delta\Delta_e/[a_1(a_1^2 - a_2^2)], \quad B = (4\Delta_e^2 - a_2^2 + a_1^2)/[2a_1(a_1^2 - a_2^2)], \\ N = \rho\delta/[a_1(a_1^2 - a_2^2)], \quad R = 2\Delta_e/(a_1^2 - a_2^2), \end{aligned}$$

$$a_{1,2} = \sqrt{(\delta^2/4 + x^2 + \rho^2 + \Delta_e^2) \pm \sqrt{(\rho^2 + \Delta_e^2 + x^2)^2 - (x^2 - \Delta_e^2)^2 - \\ - (\rho^2 + \Delta_e^2)(\rho^2 + \Delta_e^2 - \delta^2) + (\Delta_e^4 - 2\rho^2 x^2)}},$$

and  $\Delta_e = \pi n^3 r_{41} E^0 / \lambda_0$ ,  $x = (\pi n^3 p_{44} / 2\lambda_0) (2I_a / \rho v^3)^{1/2}$  ( $r_{41}$  ( $p_{44}$ ) is the electrooptical (photoelastic) constants,  $I_a$  is the ultrasonic intensity). Symbol " $\sim$ " signify  $a_1 \approx a_2$ . Detuning from Bragg condition is:  $\delta = (\pi v / f) \Delta\varphi$ , where  $\Delta\varphi = \varphi - \varphi_b$ , and  $\varphi_b = \arcsin(\lambda_0 f / 2nv)$  is Bragg angle.

We see that expression (12) indicate the existence of the elliptical polarization of the diffracted waves. When  $E^0 = 0$  the expression (12) coincide with expression for diffracted wave amplitude which has been obtained [3]. We include phase detuning coefficient  $\delta$  in expression (12) for calculation amplitude-frequency characteristics of AO devices [12].

Numerical calculations of polarization and energetic characteristics of the diffracted waves for  $\text{Bi}_{12}\text{GeO}_{20}$  crystal were carried out. We take into account that ultrasonic frequency is  $f=200$  MHz, light wave length is  $\lambda_0=0,63$   $\mu\text{m}$ . Photoelastic constant is  $p_{44}=0.04$  [13]; optical activity per unit length is  $\rho=22$  degree/mm; electrooptical constant is  $r_{41}=3,7 \times 10^{-12}$  m/V; density of crystal is  $\sigma=9,22$  g/cm<sup>3</sup>. Schematic diagram of AO interaction is presented on Fig.1. Under this conditions  $X \parallel [001]$ ,  $\vec{U} \parallel [010]$ ,  $\vec{E}^0 \parallel [010]$ .

In the Fig.2 the dependence of the diffraction efficiency  $\eta = |\vec{E}_1|^2 / |A_1|^2$  versus the interaction length  $l$  for different electric field  $E^0$  and incident light wave with s- polarization is given. Thus, when the external electric field  $E^0$  encrease the value  $\eta$  will encrease also. It should be noticed that this circumstance is due to the induced crystal anisotropy  $\Delta\epsilon_e$  which suppresses the circular anisotropy.

This physical effect may be due to the destruction of the periodical dependence the diffraction efficiency  $\eta$  from interaction length  $l$  under which full energetic exchange between zero- and first-order diffracted waves is take place [6].

The dependence of the diffraction efficiency  $\eta$  versus angle detuning coefficient are plotted in Fig.3. We see that angular sensitivity may be strongly changed by means of external electric field  $E^0$ .

Additionally, we note that for the  $\Delta\varphi=0$  the center minimum is disappeared, but the diffraction efficiency under the applied electric field are involved in all angle of light incidence.

We see from Fig.4 the dependence of the diffraction efficiency versus polarization azimuth incident light  $\varphi = \arctg(A_1/A_2)$  under the

different external electric field strain  $E^0$ . We see from Fig.4 that the diffraction efficiency did not depend from light incident polarization azimuth. The diffracted light ellipticity  $\tau = \text{tg}\{0.5 \arcsin[2 \text{Im}(q) / (1 + |q|^2)]\}$ , where  $q = (\vec{E}_1 \vec{e}_2) / (\vec{E}_1 \vec{e}_1)$ . We see on Fig.5 the dependence of  $\tau$  from  $\phi$ . Also, we see the periodic character the diffracted light ellipticity from the polarization azimuth  $\phi$ .

It should be noticed that diffracted light polarization azimuth  $\psi = 0.5 \arctg[2 \text{Re}(q) / (1 - |q|^2)]$ , i.e. polarization ellipse big axis orientation of diffracted wave, did not coincide with polarization azimuth of incidence one.

Thus, electrically-induced anisotropy has a big influence on the acoustooptical interaction in gyrotropic cubic crystal. The influence of the external electric field in the case of diffraction by the shear acoustical wave under the longitudinal electrooptical effect allowed us to have possibility to exclude any bounding by the diffraction efficiency in gyrotropic cubic crystals and reaching practically full energetic exchange between diffracted wave.

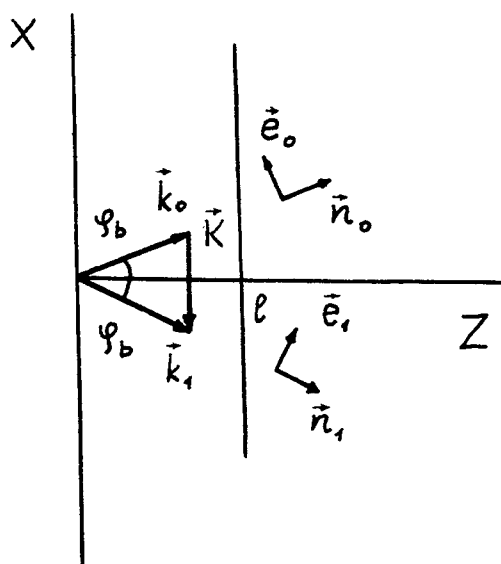


Fig.1. Schematic diagram of AO interaction (  $\vec{n}_0 = \vec{k}_0 / |\vec{k}_0|$ ,  $\vec{n}_1 = \vec{k}_1 / |\vec{k}_1|$  )

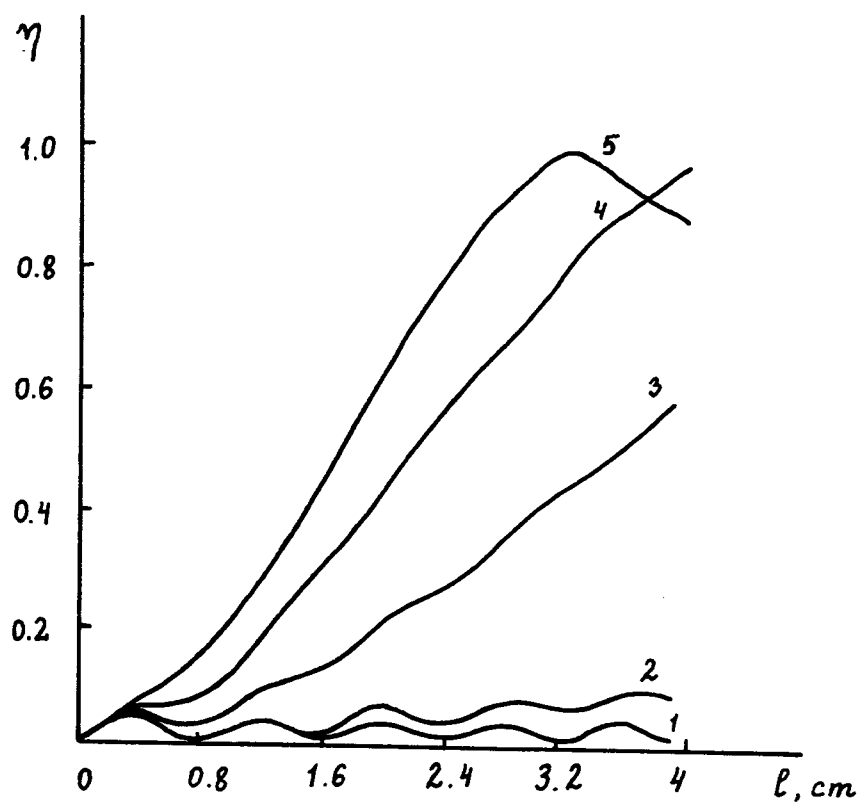


Fig.2. The dependence of the diffraction efficiency on the interaction length  $l$  for different electric field  $E^0$ : 1-0; 2-1; 3-3; 4-5; 5-7 (kV/cm).



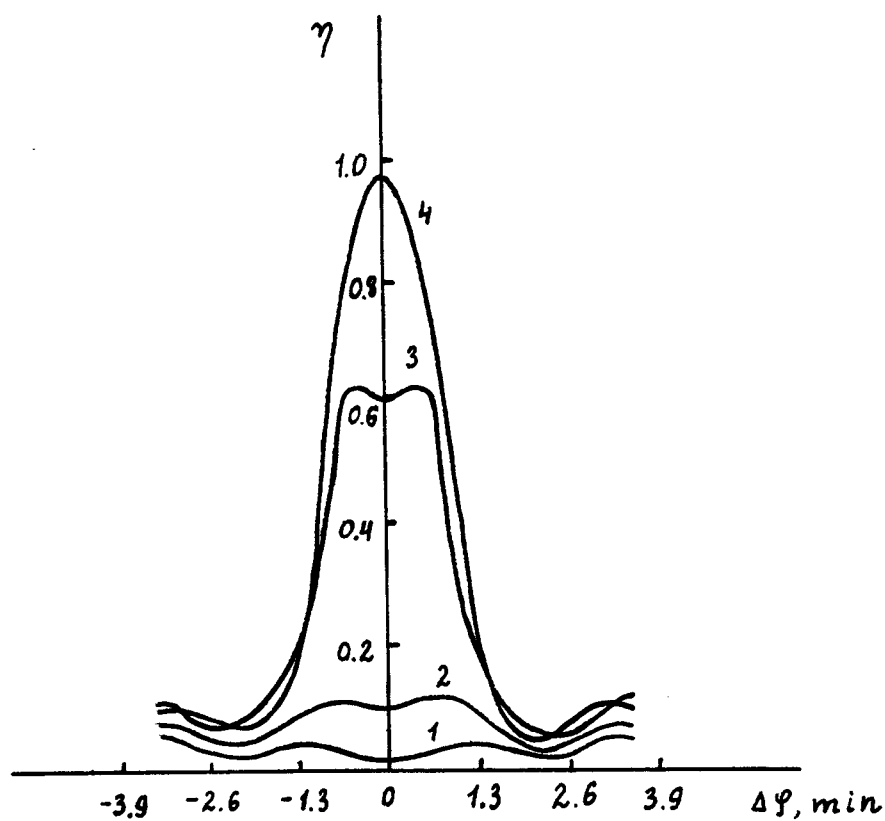


Fig.3. The dependence of diffraction efficiency  $\eta$  versus angle  $\Delta\varphi$  for different electric field  $E^0$ : 1-0; 2-1; 3-3; 4-5 (kV/cm) ( $l=4$  cm,  $I_a=10^2 \text{ W/cm}^2$ ,  $\phi=0$ ).

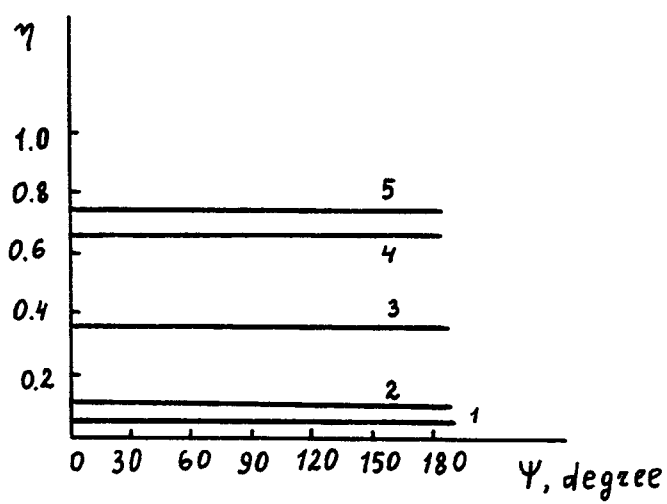


Fig.4. The dependence of diffraction efficiency  $\eta$  versus polarization azimuth  $\psi$  for different electric field  $E^0$ : 1-0; 2-1; 3-2; 4-3; 5-5 (k

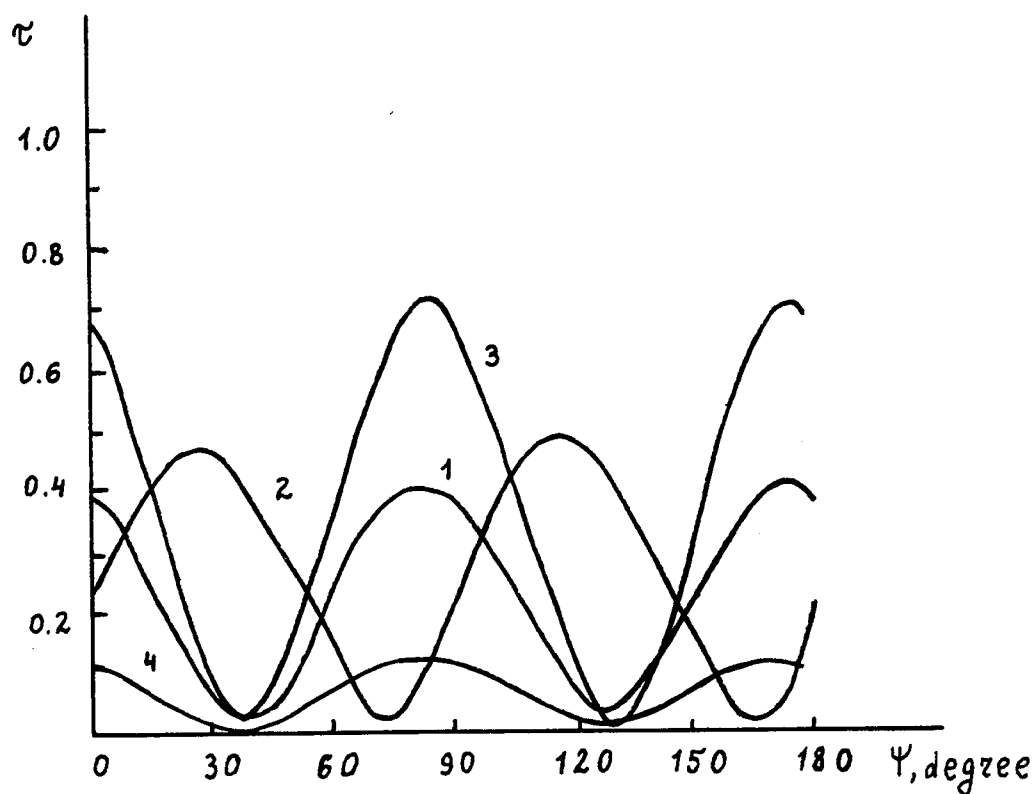


Fig.5. The dependence of ellipticity  $\tau$  versus  $\psi$  for different electric field  $E^0$ : 1-1;2-3;3-5;4-7 (kV/cm) ( $l=4$  cm,  $I_a=10^2$  W/cm<sup>2</sup>,  $\delta=0$ ).

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# Experimental Aspects of Microwave Chirality Research

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## ABSTRACT

In this paper, the focus is on experimental microwave chiral research. After briefly discussing wave propagation in a chiral medium, as well as basic equations relative to reflection and transmission through a chiral slab, and reflection from a metal-backed chiral layer, current artificial chiral materials active at microwave frequencies are mentioned. In particular, we describe the processing of chiral composites with ferroelectric ceramic inclusions. Then, the free-space measurement bench used for characterizing the samples is described, along with the method for computing the constitutive parameters. Results on the chirality parameter and the reflectivity are given. We also introduce some work done on numerical modeling with finite-element computations.

## 1. WAVE PROPAGATION, REFLECTION AND TRANSMISSION

The description of chiral, or reciprocal bi-isotropic media, requires a third complex scalar constitutive parameter in addition to the permittivity and the permeability. One way of writing the constitutive equations, known as the Drude-Born-Fedorov formalism, is as follows [1]

$$\begin{aligned}\mathbf{D} &= \epsilon_D \mathbf{E} + \epsilon_D \beta \nabla \times \mathbf{E} \\ \mathbf{B} &= \mu_D \mathbf{H} + \mu_D \beta \nabla \times \mathbf{H}\end{aligned}\quad (1)$$

Analysis of wave propagation in a chiral medium [2] shows that there are 2 canonical left- and right-circularly (LCP and RCP) polarized waves in the medium, which respective wavenumbers  $k_-$  and  $k_+$  are

$$k_- = \frac{k}{1 - k\beta} \quad \text{and} \quad k_+ = \frac{k}{1 + k\beta} \quad (2)$$

The intrinsic wave impedance of such a medium is given by  $\eta = (\mu/\epsilon)^{1/2}$ .

Using the impedance and the two LCP and RCP wavenumbers, reflection and transmission through a chiral slab of thickness  $d$  can be easily quantified. Writing

$$\Gamma = \frac{\eta - \eta_0}{\eta + \eta_0}, \quad \Phi = e^{i(k_- - k_+)d}, \quad \text{and} \quad \Psi = e^{2ikd} \quad (3)$$

where  $\eta_0$  is the impedance of free-space, the normal incidence reflection coefficient for a linearly polarized plane wave can be written as

$$S_{11} = \frac{\Gamma(1 - \Phi\Psi)}{1 - \Gamma^2\Phi\Psi} \quad (4)$$

while the normal incidence transmission coefficient (corresponding to the copolarized part of the transmitted field) is

$$S_{21} = \frac{(1 + \Phi)\Psi^{1/2}(1 - \Gamma^2)}{2(1 - \Gamma^2\Phi\Psi)} \quad (5)$$

Because the two complex LCP and RCP wavenumbers are different, both circular birefringence and dichroism occur in transmission. The transmitted wave is therefore generally elliptically polarized with an ellipticity  $\tan\phi$ , the major axis of the polarization ellipse being tilted with respect to the incident wave polarization direction by an angle  $\alpha$ . The rotation angle and the ellipticity are given by

$$\alpha = \frac{\text{Re}(k_- - k_+ )d}{2} \quad \text{and} \quad \tan\phi = \text{th}\left(\frac{\text{Im}(k_- - k_+ )d}{2}\right) \quad (6)$$

The normal incidence reflection coefficient  $R$  of the same metal-backed chiral layer can also be computed and is expressed as

$$R = \frac{(\Gamma + \Phi\Psi)}{1 - \Gamma\Phi\Psi} = \frac{\frac{\eta - \eta_0}{\eta + \eta_0} - e^{2ik_{eq}d}}{1 - \frac{\eta - \eta_0}{\eta + \eta_0} e^{2ik_{eq}d}}, \quad \text{where} \quad k_{eq} = \frac{k_- + k_+}{2} = \frac{\omega\sqrt{\epsilon_D\mu_D}}{1 - (\omega\sqrt{\epsilon_D\mu_D}\beta)^2} \quad (7)$$

All these expressions strictly apply to homogeneous or effectively homogeneous chiral media. For composites which inclusions are not necessarily small with respect to the wavelength in the matrix, they still allow a simple description of the interaction between the wave and the composite chiral medium, and yield reasonably good results [3].

## 2. ARTIFICIAL MICROWAVE CHIRAL MATERIALS

Lindmann was a pioneer in fabricating and measuring chiral composites optically active in the GHz range [4]. Recently, extensive experimental studies were carried out in the USA on composites with metallic helices [5]. Some activity is now developing in Europe. In order to fully benefit from the degrees of freedom brought by the nature of the inclusion material, it seemed interesting to us to explore the possibilities offered by ceramics. We therefore decided to work with ceramic inclusions.

A high dielectric constant material, barium strontium titanate (BST), was chosen as the inclusion material [3]. Ceramic helices are produced by coating carbon fibers with a ceramic slurry, and winding the subsequent fibers on a graphite rod. After a suitable heating cycle, carbon- and organics-free sintered ceramic helices are obtained. The helices are then randomly dispersed into an epoxy matrix, which losses can be adjusted by the addition of carbon powder.

## 3. MEASUREMENT TECHNIQUE AND MATERIAL PROPERTIES COMPUTATION METHOD

The stage following the composites processing is the evaluation of their microwave properties. A free-space measurement setup existing at IRCOM is used for that purpose. It consists mainly of 2 spot-focused horn lens antennas linked via transitions and coaxial cables to an HP 8510 network analyzer. A view of the bench is presented in figure 1. A description of its general features and calibration has already been given elsewhere [5,6], so that we should focus only on material properties measurement.

Three independent complex quantities must be obtained in order to be able to fully characterize a chiral sample. In our case, the technique used is basically a reflection-

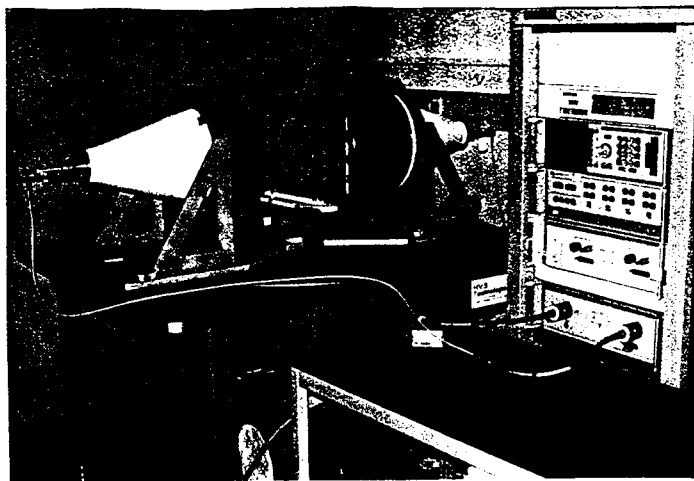


Figure 1: IRCOM free-space microwave bench. Recent upgrades make testing from 6 to 110 GHz possible. A furnace with transparent windows can be used for working up to 800°C.

transmission one (therefore thickness resonances may perturb the results). The normal incidence reflection coefficient, along with the normal incidence transmission coefficients for the copolarized and crosspolarized parts of the transmitted field are measured. The computational scheme is as follows (see [3] for details). Using (5) along with the corresponding equation for the crosspolarized part of the transmitted field, the rotation angle and the ellipticity can be directly obtained. Then, using (4), the wave impedance is computed. The next stage is the computation of  $k_-$  and  $k_+$ , where an ambiguity relative to the determination of the real parts of the 2 wavenumbers has to be solved. Finally, from the values of  $\eta$ ,  $k_-$  and  $k_+$ , those of  $\epsilon$ ,  $\mu$ , and  $\beta$  are easily deduced.

#### 4. SOME EXPERIMENTAL RESULTS

Figure 2 represents the measured chirality parameter of one of our samples. The resonance region, not represented in the figure, is around 6 GHz. At 10 GHz, the magnitude

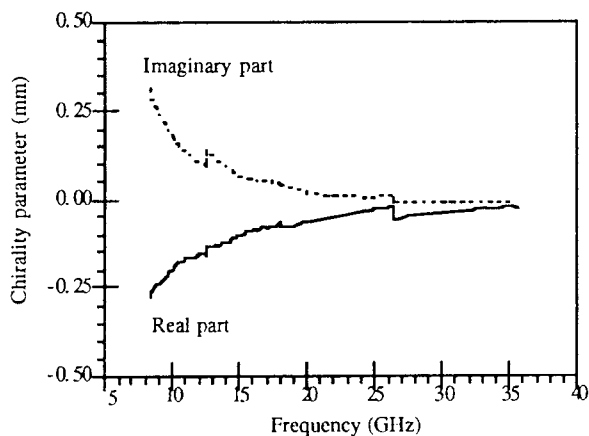


Figure 2: Chirality parameter of a composite with 3.4 % (in volume) BST helices (diameter = pitch = 3 mm, 3 turns, left-handed) in an epoxy-carbon matrix. Thickness 9.1 mm.

of  $\beta$  is about 0.28 mm: this yields values of  $|k\beta|$  and  $|k\beta|^2$  respectively equal to 0.14 and 0.02, which show that the degree of chirality is fairly low, even though the rotation angle of the composite is about  $40^\circ$  at the same frequency.

On the same sample,  $\epsilon$  and  $\mu$  were measured and the 3 constitutive parameters were used for computing the normal incidence reflectivity using equation (7). The reflection coefficient with metal backing was also measured. The comparison between computed and experimentally determined values is shown in figure 3.

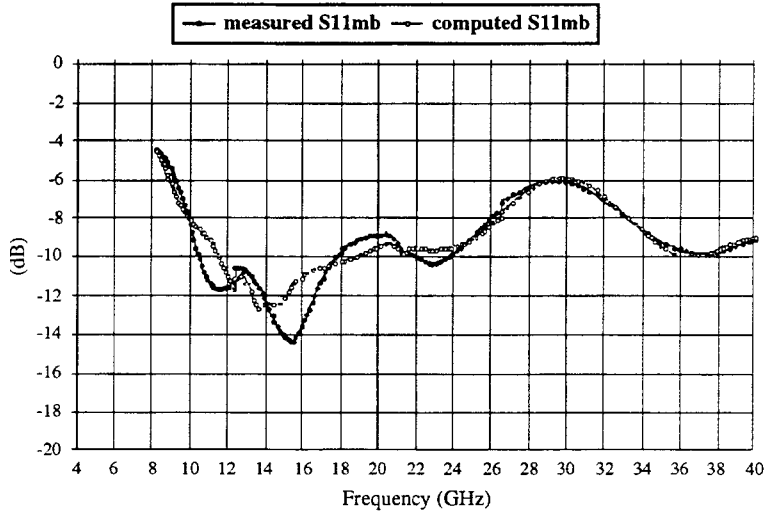


Figure 3: Measured and computed normal incidence reflectivities of a composite with 3.4 % (in volume) BST helices (diameter = pitch = 3 mm, 3 turns, left-handed) in an epoxy-carbon matrix. Thickness 9.1 mm.

The fairly good agreement between the 2 curves shows that the measured values of the constitutive parameters may be used for predicting reflectivity values, even though the size of the helices is comparable to the wavelength in the matrix and to the thickness of the sample. Further experimental work is underway to quantify scattering effects in such materials.

## 5. NUMERICAL MODELING

One of the challenges of chirality research is to establish a link between the microscopic properties of the material (i.e. the properties of the inclusions) and the macroscopic properties of a composite. We have developed a numerical technique, based on the computation of the field scattered by an helix, aimed at calculating the properties of a chiral homogeneous medium. This technique involves a finite-element computer code, Antenna Design, developed at Thomson-CSF Radars and Countermeasures Division. Preliminary results on the radar cross section of ceramic helices have been presented recently [7]. More detailed results on metallic and dielectric inclusions, as well as experimental validation of the modeling results will be reported soon.

## CONCLUSION

In this paper, we tried to give a panorama of microwave chirality research at Thomson-CSF and IRCOM. Ceramic helices are fabricated by a coating-winding technique, which allows a wide range of materials to be shaped in the form of a small size spring, and dispersed



in a host medium to produce chiral composites. Using a free-space bench, normal incidence measurements are carried out on the materials and their effective properties, including chirality, are computed by an analytical method. Because of the heterogeneous character of microwave chiral composites, it appears that new measurements and numerical models are necessary to better quantify propagation and loss phenomena in these materials. Ongoing work is currently devoted to computer simulation of electromagnetic scattering by an helix.

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# THE COMPOSITE GYROTROPIC PLATES

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In the development of works [1,2] we have dealt with the plate composed from two equal gyrotropic crystal plates. The angle between their principal directions (their fast axes) is  $\theta$ .

We assume that the eigen waves are orthogonal (one to another), their ellipticities  $k_{1,2} = \tan \gamma_{1,2}$  are equal, the reversal of the ellipses is an opposite, i.e.  $k_2 = -k_1$  in the each plate.

We select the fast axis of the entrance plate as the azimuth reference. We denote it  $V$ . Then the exit plate azimuth is  $\theta$ . Later on fast axes directions of entrance and exit plates we shall call the entrance axis and the exit axis respectively. To analyze the action of such composite plate at the passed light we have written the Mueller matrix of the composite plate:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2AB\sin 2\theta - (A^2 - B^2 - D^2)\cos 2\theta & 2AB\cos 2\theta + (A^2 - B^2 + D^2)\sin 2\theta & -2D(A\cos 2\theta - B\sin 2\theta) \\ 0 & -2AB\cos 2\theta - (A^2 - B^2 - D^2)\sin 2\theta & 2AB\sin 2\theta - (A^2 - B^2 + D^2)\cos 2\theta & -2D(A\sin 2\theta + B\cos 2\theta) \\ 0 & -2AD & 2BD & 1 - 2D^2 \end{bmatrix} \quad (1)$$

where  $A = [1 - \sin^2 2\gamma (1 - \cos \delta)] \sin \theta - \sin 2\gamma \sin \delta \cos \theta$ ;  $B = \cos \delta \cos \theta + \sin 2\gamma \sin \delta \sin \theta$ ;  $D = \cos 2\gamma [\sin 2\gamma \sin \theta (1 - \cos \delta) + \sin \delta \cos \theta]$ ,  $\delta = 2\pi d(n_2 - n_1)/\lambda$  is the phase difference due to eigen waves passage through the each plate,  $d$  is an each plate thickness,  $n_1, n_2$  are refractive indices of eigen waves. If one introduce new notations

$$\tan 2\theta_1 = A/B = \frac{\sin \theta - \sin 2\gamma [\sin 2\gamma \sin \theta (1 - \cos \delta) + \cos \theta \sin \delta]}{\cos \theta \cos \delta + \sin 2\gamma \sin \theta \sin \delta}, \quad (2)$$

$$\sin \Delta/2 = D = \cos 2\gamma [\sin 2\gamma \sin \theta (1 - \cos \delta) + \cos \theta \sin \delta],$$

the matrix (1) may be written in the form of a two matrices product:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\varphi & \sin 2\varphi & 0 \\ 0 & -\sin 2\varphi & \cos 2\varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C^2 + S^2 \cos \Delta & CS (1 - \cos \Delta) & S \sin \Delta \\ 0 & CS (1 - \cos \Delta) & S^2 + C^2 \cos \Delta & -C \sin \Delta \\ 0 & -S \sin \Delta & C \sin \Delta & \cos \Delta \end{bmatrix} \quad (3)$$

where  $C = \cos 2\theta_1$ ,  $S = \sin 2\theta_1$ ,  $\varphi = 2\theta_1 - \theta$ .

It is easy seen that the first matrix is a turn matrix through an angle  $\varphi$ , the second one is the Mueller matrix of some birefringence nongyrotropic crystal plate. The phase difference of this plate is  $\Delta$  and its fast axis is turned through an angle  $\theta_1$  with respect to V. In order to understand the physical meaning of parameters  $\theta_1$  and  $\Delta$  we multiply (3) by the Stokes's vector of the incident light. The incident light is linearly polarized and its azimuth is  $\theta_1$ . After simple transformations it is seen that the exit light is linearly polarized too and its azimuth is  $\theta - \theta_1$  with respect to V, i.e. its azimuth is  $-\theta_1$  with respect to exit axis. Thus two axes are defined, one on the entrance face (its azimuth is  $\theta_1$  with respect to entrance axis), the other on the exit face (its azimuth is  $-\theta_1$  with respect to exit axis), of the composite plate, such that light plane polarized with its vector parallel to the incident axis emerges plane polarized with its vector parallel to the emergence axis. We shall call these axes "effective fast axes" by analogy with the plate composed from nongyrotropic plates [1]. By the same analogy  $\Delta$  is called "the effective phase difference" of the composite plate.

Let us examine exceptions of parameters  $\theta_1$  and  $\Delta$ . It is seen from (2) that  $\Delta=0$  by  $k=1$  ( $\gamma=\pi/4$ ) or by  $\theta=\theta_R$

$$\operatorname{tg} \theta_R = -1/\sin 2\gamma \operatorname{tg} \delta/2. \quad (4)$$

Really taking into account (2) and (4) we have

$$\operatorname{tg} \varphi = \operatorname{tg}(2\theta_1 - \theta) = (A \cos \theta_R - B \sin \theta_R) / (A \sin \theta_R + B \cos \theta_R) = -\operatorname{tg} 2\theta_1. \quad (5)$$

Thus if two gyrotropic plates have turned by their fast axes through the angle  $\theta_R$  determined from (4) the composite plate changes only the azimuth of the emergence light at the angle  $2\theta = 2\theta_R$ . This case corresponds to one by  $\gamma = 0$  and  $\theta = \pi/2$ .

Two gyrotropic plates may be too combined to produce circular polarized light from plane polarized one. By multiplying the Stokes's vector of the incident plane polarized light into (2) we get the normed Stokes's vector of the emergence light. By equating to 0 its last component we get:

$$M_{42} \cos 2\alpha_0 + M_{43} \sin 2\alpha_0 = 1, \quad (6)$$

where  $M_{ij}$  are elements of the matrix  $M$ ,  $\alpha_0$  is the azimuth of incident plane polarized light, plus or minus corresponds to right- or left-circular polarization on the exit face. By taking into account relationship (2) we have

$$\sin 2(\alpha_0 - \theta_1) \sin \Delta = \pm 1. \quad (7)$$

It is clear that moduli of both co-factors must be equal 1. Consequently the second relationship from (2) can write as

$$\sin \Delta/2 = \cos 2\gamma [\sin 2\gamma \sin \theta (1 - \cos \delta) + \sin \delta \cos \theta] = \pm \sin \pi/4. \quad (8)$$

One can take the angle  $\zeta$  such that

$$\begin{aligned} \cos \zeta &= \sin 2\gamma \cos 2\gamma (1 - \cos \delta) / \sqrt{1 - \cos^4 2\gamma (1 - \cos \delta)^2}, \\ \sin \zeta &= \cos 2\gamma \sin \delta / \sqrt{1 - \cos^4 2\gamma (1 - \cos \delta)^2}, \end{aligned}$$

then (8) is rewritten as

$$\sin (\zeta + \theta) = \pm (\sin \pi/4) / \sqrt{1 - \cos^4 2\gamma (1 - \cos \delta)^2}. \quad (9)$$

From (9) is followed

$$\left| \sqrt{2} [1 - \cos^4 2\gamma (1 - \cos \delta)^2]^{-1/2} / 2 \right| \leq 1. \quad (10)$$

After the transformation of (10) we get the equivalent inequality

$$(2 - \sqrt{2})/4 \leq \cos^2 2\gamma \sin^2 \delta/2 \leq (2 + \sqrt{2})/4. \quad (11)$$

It is seen that the equality (7) is not true for arbitrary values  $\theta_1$ ,  $\Delta$  and hence for arbitrary ones  $\gamma$ ,  $\delta$ .

Thus it is possible to transform the plane polarized light to the circularly polarized light by using two equal gyrotropic crystal plates. To do this one would require to be convinced that values  $\gamma$ ,  $\delta$  of taken place satisfy by inequality (11). Then it would defined sequentially values  $\theta$ ,  $\theta_1$ ,  $\alpha$  from relationships (9), (2), (7) respectively.

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# ACOUSTO-ELECTRON INTERACTION IN CONDUCTOR CRYSTAL OF FERROELECTRIC CERAMIC IN THE CONDITION OF INDUCING OF PIEZOELECTRIC, ANISOTROPIC AND GYROTROPIC PROPERTIES BY THE ROTATING ELECTRIC FIELD

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In the article [1] a possibility of rotating acoustic anisotropy forming in crystals under outside electric field influence is shown. In the book [2] is proved that the crystal placed into space-uniform rotating electric field has analogy of gyrotropic properties to crystal with stationary spiral-nonuniform acoustic anisotropy. In the article [3] the propagation of the acoustic waves in the spiral dielectric structures with great values of the permittivity controlled by the electric field is investigated. In the given work the propagation of the cross acoustic waves in conductor crystal with nonstationary anisotropy induced by the rotating electric field taking into account ultrasonic interaction with free conduction electrons is considered. The reversed and passed waves intensities dependence from thickness of crystal is studied.

We shall describe the properties of crystal of ferroelectric ceramic (for example, on the basis of barium titanatum) using the material equations

$$\sigma = c\gamma + \varepsilon_0 \varepsilon g E E_0 \quad D = \varepsilon_0 \varepsilon E - \varepsilon_0 \varepsilon g E_0 \gamma$$

where  $\sigma, \gamma, c$  - tensors of tensions, deformations and elastic constants,  $g$  - tensor of rank four,  $gE_0$  - tensor of rank three, taking into account piezoelectric effect induced by the rotating electric field,  $\varepsilon_0$  - the electric constant,  $\varepsilon$  - the relative permittivity of the medium.

The crystal is placed into electric field rotating round axis  $X$  (unit vector  $a$ ) with components

$$E_{01} = 0, E_{02} = -E_0 \sin \Omega t, E_{03} = E_0 \cos \Omega t$$

$E_0$  and  $\Omega$  - amplitude and electric field rotation frequency.

We shall describe the influence of acoustic wave on the free conduction electrons in crystal using the Maxwell's equation

$$\frac{dD}{dx} = -en$$

where D - electric induction, e - elementary electric charge, n - change of electron concentration, which is caused by the acoustic wave.

The equation of elastic wave propagation is

$$\frac{d\sigma}{dx} = \rho \frac{d^2 u}{dt^2}$$

where u - displacement vector,  $\rho$  - medium density.

We can define the movement of electrons captured by longitudinal piezoelectric field of acoustic wave by the equation of continuity

$$\frac{dj}{dx} = e \frac{dn}{dt}$$

and the equation of motion of electron

$$\frac{dv}{dt} = - \frac{e}{m^*} E - \nu v - \frac{k_b T}{m^* N_0} \frac{du}{dx} \quad (2)$$

here  $j = -eN_0 v$  - vector of density of current,  $v$  - velocity of electron,  $m^*$  - effective mass of electron,  $\nu$  - frequency of collision,  $k_b$  - Boltzmann constant,  $T$  - absolute temperature,  $N_0$  - balanced concentration of electrons.

The use of the method which was proposed in [1,4] allows to determine the wave numbers and the ellipticities of the proper modes of the crystal with rotating anisotropy.

Let's consider the case when on the crystal border in  $x=0$  circular-polarized acoustic wave is excited

$$u_e = u_0 n \exp[-i\omega_0 t + ik_0 x] \quad (3)$$

elastic displacement vector of which has the same rotation direction in time as the outside electric field.

As a result of propagation in the crystal ultrasonic waves interaction with rotating electric field, amplification of the passed wave at the frequency  $\omega_0$

$$u_\tau = u_\tau n \exp[-i\omega_0 t + ik' x] \quad (4)$$

and generation of the reversed wave at the frequency  $2\Omega - \omega_0$  can take place.

$$u_c = u_{c+} n_+ \exp[-i(\omega_0 - 2\Omega)t + ik'x] \quad (5)$$

As a result of ultrasonic waves reflection from the border of the crystal with rotating anisotropy, the reflected wave at the frequency  $\omega_0$

$$u_r = u_{r-} n_- \exp[-i\omega_0 t - ik'x] \quad (6)$$

and the passed wave at the frequency  $2\Omega - \omega_0$  can also appear

$$u_{c\tau} = u_{c\tau+} n_+ \exp[-i(\omega_0 - 2\Omega)t - ik'x] \quad (7)$$

There  $k', k''$  are wave numbers depending on waves frequencies and parameters of density and elasticity of the medium bordering when  $x=0$  and  $x=L$  with the crystal placed into rotating electric field.

Representing according to [5] the acoustic field in the crystal with rotating anisotropy in the form of the superposition of four proper modes, from the conditions of continuity of wave elastic displacement vectors (3)-(7) and continuity of tensions tensor components on the borders of the crystal [6], we have the system of eight equations.

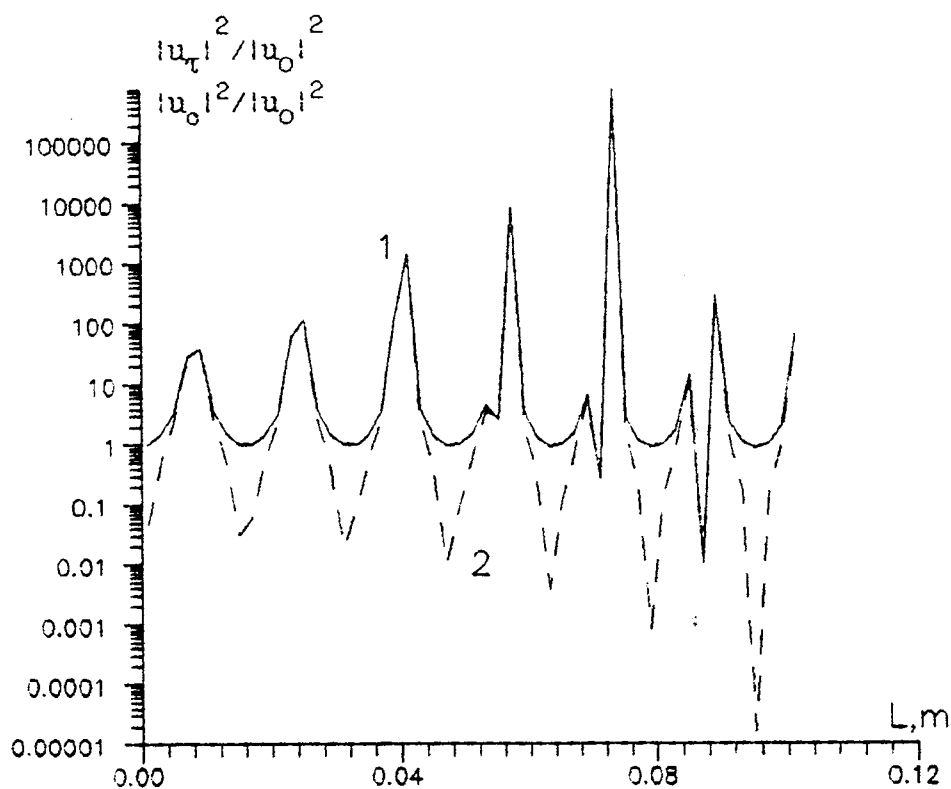
The eight equation system has a large size therefore we do not write it here.

The amplitudes of all waves can be determinated as a result of numerical solution of the eight equation system. As a example in figure is presented diagrams of reversed and passed (at the frequency  $\omega_0$ ) waves intensities dependence on the thickness of the crystal  $L$  in the case of resonance interaction, i.e. when the frequency and rotation direction of electric field coincide with those of the displacement vector of acoustic waves ( $\omega_0 = \Omega$ ). Calculations were made with the values of parameters

$v_t = 2.5 \times 10^3$  m/s - the velocity of acoustic waves without influence of electric field,  $m^* = 0.0145m_e$ ,  $\Omega = 10^9$  Hz,  $\omega_0 = 10^9$  Hz,  $\nu = 10^{13}$  s $^{-1}$ ,

$T = 290$  K,  $c = 10^{11}$  N/m $^2$ ,  $\rho = 5.7 \times 10^3$  kg/m $^3$ ,

the change of parameters, which is caused by the electric field has range a few per cent.



The passed at frequency  $\omega_0$  and the reversed waves intensities dependence on the thickness of the crystal with rotating anisotropy

$$1 - |u_\tau|^2, \quad 2 - |u_0|^2$$

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# DISSIPATIVE PROPERTIES OF GYROTROPIC SUPERLATTICES IN THE LONG WAVELENGTH APPROXIMATION

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The light energy dissipation in gyrotropic superlattices (SL) is calculated in the approximation for the optical wavelengths  $\lambda \gg D$ , where  $D$  is the SL period. The determination of the SL parameters by photothermoacoustic methods is proposed.

SL optical properties are simply described in the long wavelength approximation (LWA) when the period of the SL is more less the lengths of optical waves propagating in the superstructure. Then we can consider the SL as a homogeneous medium characterized by a set of the effective parameters [1-3].

In this paper the dissipation of the electromagnetic field energy is investigated in the LWA for the SL including nonmagnetic crystals of the cubic symmetry. The one-dimensional consideration is made with taking into account the multibeam optical interference and circular dichroism of the SL components.

## 1. Theoretical model.

We assume a monochromatic elliptically polarized light incident on the SL normally to the layers boundaries at the plane  $z=0$  (in the SL region  $0 \leq z \leq 1$ ). The SL consisting of absorbing cubic crystals is characterized by the axially symmetric complex dielectric constant tensor  $\epsilon_e$  and optical activity tensor  $\gamma_e$  [3]. The equal principal values of these tensors are:

$$\begin{aligned} (\epsilon_e)_{11} &= (\epsilon_e)_{22} = x\epsilon_1 + (1-x)\epsilon_2, \\ (\gamma_e)_{11} &= (\gamma_e)_{22} = x\gamma_1 + (1-x)\gamma_2. \end{aligned} \quad (1)$$

Here the period  $D$  of the SL consists of two layers with relative thicknesses  $x=d_1/D$  and  $1-x=d_2/D$  ( $d_1+d_2=D$ ). The quantities with indexes "e,1,2" concern the effective medium, first and second component of the SL correspondingly. Circular dichroism is described by imaginary parts of the optical activity tensors which will be designated  $\gamma_e'', \gamma_1'', \gamma_2''$ .

Optical properties of the axially symmetric gyrotropic crystal in the direction of the optical axis are equivalent to the ones for the optically active isotropic medium with the

complex parameters  $\varepsilon_e = (\varepsilon_e)_{11}$ ,  $\gamma_e = (\gamma_e)_{11}$  [4]. So the dissipation of the energy in SL can be described by the familiar relations [5,6], with taking into account Eqs.(1):

$$Q_e = Q_+ + Q_- , \quad (2)$$

$$Q_{\pm} = N_0 I_0 \alpha_{\pm} [N_+ T_{\pm} \exp(-\alpha_{\pm} z) + N_- T_{\mp} \exp(\alpha_{\pm} z - 2\beta l)] ,$$

where  $N_0 = n' n_1^2 / \xi$ ,  $N_{\pm} = |n_0 \pm n_2|^2$ ,  $T_{\pm} = (1 \pm \tau)^2 / (1 + \tau^2)$ ,  $\beta = (4\pi/\lambda) n_0''$ ,  $\alpha_{\pm} = (4\pi/\lambda) (n_0'' \pm \gamma_e'')$ ,  $\xi = \xi_1 + [\xi_2 \sin(\alpha l) + \xi_3 \cos(\alpha l)] \exp(-\beta l) + \xi_4 \exp(-2\beta l)$ ,  $\alpha = (4\pi/\lambda) n_0'$ ,  $\xi_1 = |n_0 + n_1|^2 N_+$ ,  $\xi_2 = 4n_0'' (n_1 + n_2) (|n_0|^2 - n_1 n_2)$ ,  $\xi_3 = 8n_1 n_2 n_0''^2 - 2(|n_0|^2 - n_1^2) (|n_0|^2 - n_2^2)$ ,  $\xi_4 = |n_0 - n_1|^2 N_-$ . Here  $I_0$  and  $\tau$  are incident light intensity and ellipticity ( $\tau \leq 0$  at left polarization),  $n_0 = \sqrt{\varepsilon_e} = n_0' + i n_0''$  ( $i^2 = -1$ ), and quantities with indexes "+" correspond to the left and right circular polarized waves superposition of which describes the field in the effective medium. We assume non-absorbing media behind and in front of the SL to have real refractive indexes  $n_2$  and  $n_1$  correspondingly.

Eqs.(2) are rather complicated for the analysis. Even neglecting the SL components dichroism and reflected waves we obtain the equation of degree 5/2 from the one  $dQ_e/dx=0$ . At  $l \gg 1/\beta$  Eqs.(2) are simplified

$$Q_{\pm} \approx N_0 I_0 \alpha_{\pm} N_{\pm} T_{\pm} \exp(-\alpha_{\pm} z), \quad \xi = \xi_1 , \quad (3)$$

that corresponds to the semi-infinite SL case.

It is seen from Eqs.(2) that described by the quantity  $\xi$  multibeam interference takes effect at  $l \leq 1/\beta$ . In this case at usual assumptions  $\sqrt{\varepsilon_e}'$ ,  $n_1$ ,  $n_2$ ,  $\sqrt{\varepsilon_e}' + n_1$ ,  $\sqrt{\varepsilon_e}' + n_2 \gg \varepsilon_e'' / (2\sqrt{\varepsilon_e}')$  Eqs.(2) give

$$\xi = \xi_+^2 + \xi_-^2 - 2\xi_+ \xi_- \exp(-\beta l) \cos(\alpha l) , \quad (4)$$

where  $\xi_{\pm} = (a \pm n_1)(a \pm n_2)$ ,  $a = \sqrt{\varepsilon_e}'$ . So the optical interference effect on the SL dissipation is characterized by the parameter  $\cos(\alpha l)$ , where  $\alpha = (4\pi/\lambda) [x\varepsilon_1' + (1-x)\varepsilon_2']^{1/2}$

To compare the SL and its components dissipative properties we used the parameters:  $\eta_j = Q_e / Q_j$ ,  $\rho_j = \Delta Q_e / \Delta Q_j$ ,  $j=1,2$ , where  $\Delta Q_j = Q_j(+\tau) - Q_j(-\tau)$ ,  $i=e,1,2$  and  $Q_1 = Q_e|_{x=1}$ ,  $Q_2 = Q_e|_{x=0}$ . Here  $\eta_j$  characterises the dissipation and  $\rho_j$  - the difference in dissipation for the right and left polarized light in the SL relatively to the same quantities in the SL component  $j$  (at  $z=\text{const}$ ).

## 2. Graphical analysis and discussion.

The following quantities were assigned constant values:  $I_0 = 0.15 \text{ W/sm}^2$ ,  $n_1 = 1$ ,  $n_2 = 1.5$ ,  $z = 1 \mu\text{m}$  (bright limits varying of  $z$  did

not change the form of the dependencies reported here). The parameters  $l, x, \lambda, \tau$  and SL components properties were changed. The  $Q_e(x)$  dependence at various parameters (Tab.1,  $\gamma_1''=10^{-5}, \gamma_2''=3 \cdot 10^{-5}$ ;  $\lambda=0.55 \mu\text{m}$ ,  $\tau=1$ ) is illustrated by Fig.1. The  $Q_e(x)$  form mainly described by Eqs.(1) is near linear and symmetric at the transposition of layers  $1 \leftrightarrow 2$  (curv.1,2).  $Q_e(x)$  oscillates with parameters  $\lambda$  at  $l \leq 1/\beta$  (curv.3). At the data the SL dissipation have practically no dependence on the components mass parts (curv.4).

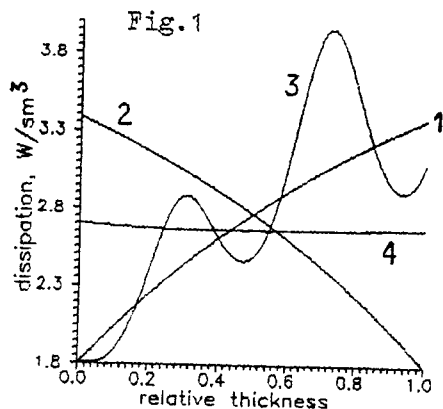


Table 1.

curve	$\varepsilon_1$	$\varepsilon_2$	$l, \mu\text{m}$
1	$(7,3 \cdot 10^{-2})$	$(4,10^{-2})$	60
2	$(4,10^{-2})$	$(7,3 \cdot 10^{-2})$	60
3	$(7,3 \cdot 10^{-2})$	$(4,10^{-2})$	1
4	$(5,1.8 \cdot 10^{-2})$	$(2,10^{-2})$	60

The  $\eta_2(\lambda)$  dependence at  $\varepsilon_1=(3,1.5 \cdot 10^{-2})$ ,  $\varepsilon_2=(5,2 \cdot 10^{-2})$ ,  $x=0.2$ ,  $l=3 \mu\text{m}$  (curv.1),  $40 \mu\text{m}$ (2), and the same values of  $\gamma_1'', \gamma_2'', \tau$  is shown in Fig.2. One can note a characteristic beats form well described by Eq.(4) and that  $\eta_2 > 1$  at the definite  $\lambda$  (though here  $\beta_1 < \beta_e < \beta_2$  for absorptivities). It is interesting that  $\rho_j(\lambda)$  dependencies are practically the same shown in Fig.2. So at the definite parameters the SL dissipative properties including dichroic ones will not be intermediate between the same components properties. Strong oscillations of light absorption in the SL relatively to absorption in the components appear at  $l \leq 1/\beta$ .

Fig.2

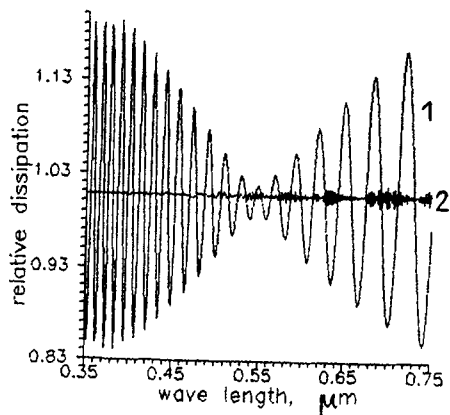
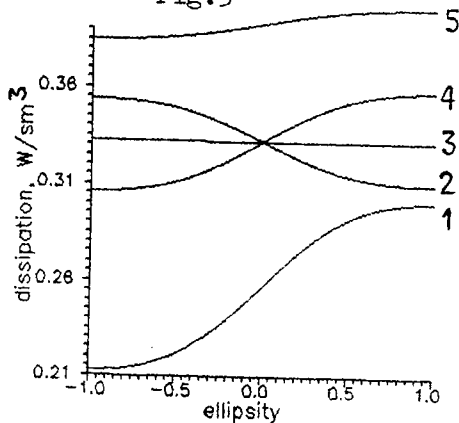


Fig.3



The effect of incident light ellipticity on the gyrotropic SL dissipation is characterized by Fig.3. Here  $\varepsilon_1 = (3, 6 \cdot 10^{-3})$ ,  $\varepsilon_2 = (5, 10^{-3})$ ,  $\gamma_1'' = 6 \cdot 10^{-5}$  (curv. 1, 4, 5),  $-6 \cdot 10^{-5}$  (2, 3),  $\gamma_2'' = 5 \cdot 10^{-6}$  (1, 2, 4, 5),  $6 \cdot 10^{-5}$  (3),  $l = 5 \mu\text{m}$ ,  $\lambda = 0.55 \mu\text{m}$ ,  $x = 0.1$  (5),  $0.5$  (2, 3, 4),  $0.9$  (1). The weak  $Q_g(\tau)$  dependence at  $\gamma_g'' \leq 10^{-6}$  with growth of the  $\gamma_g''$  becomes non-linear (1, 4, 5) especially at near-circular polarization. The data of Fig.3 show too that variation of the geometry and optical constants of the components gives the opportunity to gain the SL with designed dichroic properties (2, 3, 4).

The data reported can be used for the control and determination of the SL parameters by photothermoacoustic methods [7] where the signal measured is proportional to the value of absorbed light energy. For example, as it is seen from Eqs.(1) and Fig.1 when  $x = 0.5$  the signal must not change at the radiation from the SL opposite sides (with taking into account the backing effect). At arbitrary  $x$  having determined the wavelengths for two neighbour maxima of the  $Q_g(\lambda)$  one can gain from Eq.(4) with taking into account the dispersion  $\varepsilon_1(\lambda)$ ,  $\varepsilon_2(\lambda)$  the quadratic equation in the unknown  $x$ . At the known  $x$  the SL components optical constants can be determined.

So at typical parameters the simple model advanced predicts some characteristic dissipative properties of the gyrotropic SL satisfying the long optical wavelength approximation.

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# SYMMETRY OF TENSORS AND OPTICAL PROPERTIES OF DIRECTIONS IN MAGNETICAL CRYSTALS

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Syrotin and Shaskolskaya [1], Zheludev [2] have introduced the concept of optical properties of directions (OPD) in crystals. OPD are polarizable characteristics and birefringences of own plane monochromatic waves in these directions.

We use Maxwell equations for plane monochromatic waves and material equations [3] for vectors of electromagnetic field  $\underline{E}$ ,  $\underline{D}$ ,  $\underline{B}$ ,  $\underline{H}$  to description of optical properties of crystals.

$$\underline{E} = \varepsilon^{-1} \underline{D} + \alpha \underline{H}, \quad \underline{B} = \mu \underline{H} + \beta \underline{E}, \quad (1)$$

These equations describe various types of anisotropy, gyrotropy and absorption of linear media. As for transparent media

$$\varepsilon = \varepsilon^+, \quad \mu = \mu^+, \quad \alpha = -\beta^+, \quad (2)$$

sign "+" means Ermit's conjugate.

According to Maxwell equations and (1) we may obtain the wave equation [4].

$$\underline{M} \underline{D} = [I(\varepsilon^{-1} + \alpha \underline{n} \alpha^+ - 1/n^2)I + 1/n(I \alpha \underline{n}^\times - \underline{n}^\times \alpha^+ I)] \underline{D} = 0 \quad (3)$$

where  $I = -\underline{n}^\times \underline{n}^\times$ ,  $\mu = 1$ ,  $\underline{n}^\times$  - antisymmetrical tensor of second rank which is dual to vector of wave normal  $\underline{n}$  ( $\underline{n}^2 = 1$ ). Symmetry of matrix  $\underline{M}$  defines symmetry of OPD along  $\underline{n}$  selected.

It is possible to choose parts in material tensors  $\varepsilon^{-1}$  and  $\alpha$ . These parts answer for various optical effects.

$$\varepsilon^{-1} = \chi + i \underline{G}^\times, \quad \alpha = i \alpha_{oa} + \alpha_{me}, \quad (4)$$

where  $\chi$  is symmetrical  $i$ -tensor of second rank describing linear birefringence,  $\underline{G}$  - vector of magnetical gyration, characterizing Faraday's effect. Tensor  $\alpha_{oa}$  - axial nonsymmetry  $i$ -tensor of second rank, defining natural optical activity,  $\alpha_{me}$  - axial nonsymmetric  $c$ -tensor of the second rank describing magnetoelectrical effect.

c-tensor of the second rank describing magnetoelectrical effect.

It is not necessary to solve (3) for qualitative analyses of symmetry OPD.

That is why  $\mathbb{N}$  may be subdivided to items and be looked for their symmetry. Every item corresponds to definite optical effect.

We released research all classes of magnetical symmetry so that should know availability those optical effects along different crystallographic directions.

For example we gave the tables of optical properties of cubic and uniaxial crystals. In tables signs "+" or "-" correspond to the presence or the absence of the effects, which are interesting for us.

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Table 1

OPD in cubic crystals  
n-arbitrary

Classes	BR	MEE	OA	PE
432, 23	-	+	+	-
$m\bar{3}'m$ , $m3m$ , $m3m'$ , $m'3m$ , $\bar{4}3'm'$ , $\bar{4}3m$ , $m3'$ , $m3$	-	-	-	-
$\bar{4}3m'$ , $m'3$ , $m'3m'$	-	+	-	-
43'2, 4'32, 23'	-	-	+	-

Table 2

OPD in uniaxial crystals  
 $n_c \neq 0$ ;  $[n_c] \neq 0$

Classes	BR	MEE	OA	PE
$\bar{4}$ , 4, 6, $\bar{4}2'm'$ , $42'2'$ , $62'2$ , $32'$ , 3, $\bar{6}$	+	+	+	+
$6/m'mm$ , $6/m'm'm'$ , $4/m'm'm'$ , $4/m'mm$ , $4'/m'mm'$ , $4mm$ , $6mm$ , $3m$ , $4mm'$ , $\bar{6}'m2'$ , $\bar{3}'m$ , $4/m'$ , $6/m'$ , $4'/m'$ , $\bar{6}'$ , $\bar{3}'$ , $\bar{6}'m'2$ , $\bar{3}'m'$	+	+	-	-
$\bar{4}2m$ , $\bar{4}'2'm$ , $\bar{4}'2m'$ , $\bar{4}'$ , $4'$ , $3'$ , $4'22$ $422$ , $622$ , $32$	+	+	+	-
$4m'm$ , $6m'm'$ , $3m'$	+	+	-	+
$6/mmm1'$ , $6/mmm$ , $6'/mmmm'$ , $6'/m'mm'$ $4'/mmmm1'$ , $4'/mmmm$ , $4'/mmmm'$ , $4mm1'$ , $6mm1'$ , $3'm$ , $\bar{6}m21'$ , $\bar{6}m2$ , $\bar{6}'mm'$ , $\bar{3}'m1'$ , $\bar{3}m$ , $6/m1'$ , $6'/m$ , $6'/m'$ , $4/m1'$ , $4'/m$ , $\bar{6}1'$ , $\bar{3}1'$	+	-	-	-
$\bar{4}2m1'$ , $4221'$ , $6221'$ , $3'2$ , $6'22'$ , $\bar{4}1'$ , $41'$ $61'$ , $6'$	+	-	+	-
$4/m$ , $6/m$ , $6/mmm'm'$ , $4/mmm'm'$ , $\bar{6}m'2'$ , $\bar{3}$ , $\bar{3}m'$	+	-	-	+

## BIANISOTROPICS OF QUASICRYSTALS. SYMMETRY ASPECTS.

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Symmetry of physical tensor quasicrystal properties has been discussed. Rules of selection for tensors of natural optical activity for different classes of quasicrystal symmetry have been determined.

Recently quasicrystal-anisotropic solids with infracted translational symmetry have been discovered by experiment [1]. There are "prohibited" axes of 5, 8, 10 and 12 fold symmetry in them. The paper seeks to carry out symmetrical analysis of quasicrystal optical properties.

For phenomenological description of macroscopic physical properties of quasicrystals in accordance with Neuman principle [2] we will use the following point symmetry groups: 5, 5/2,  $\bar{5}$ , 5m, 532, m5m, 8, 8/2,  $\bar{8}$ , 8mm, 8/m,  $\bar{8}2m$ , 8/mmm, 10, 10/2,  $\bar{10}$ , 10mm, 10/m,  $\bar{10}2m$ , 10/mmm, 12, 12/2,  $\bar{12}$ , 12mm, 12/m,  $\bar{12}2m$ , 12/mmm.

By way of example consider the linear optical properties of quasicrystals.

It should be kept in mind that due to high symmetry of quasicrystals many properties characterized by 1-4 th rank tensors will be uniaxial and even isotropic [3,4]. It follows from hermann theorem [2], according to which axis of n-fold symmetry for  $r < n$  rank tensor would be also an axis of infinite order.

That's why symmetrical tensors of rank 2 of dielectric and magnetic permeability in media of classes 532 and m5m would be isotropic and in the rest classes of quasicrystals would be unia-



xial.

By natural optical properties (NOA), characterized by nonsymmetric pseudotensor of rank 2, quasicrystals are divided into 5 types. Symmetry classes 5, 8, 10, 12 are characterized by three independent components:  $11=22$ ,  $33$ ,  $21=-12$ . In groups  $5/2$ ,  $8/2$ ,  $10/2$ ,  $12/2$  are nonzero components  $11=22$ ,  $33$ . In quasicrystals of symmetry:  $5m$ ,  $8mm$ ,  $10mm$ ,  $12mm$  NOA tensor is purely antisymmetric:  $21=-12$ . In the samples of symmetry  $532$  NOA is isotropic. The rest point groups of quasicrystals symmetry prohibit NOA.

Due to frequency dispersion physical properties depend on the frequencies. That's why there are frequency regions of electromagnetic radiation, where optical properties are characterized by tensors  $\epsilon$  and  $\mu$ , i.e. quasicrystal is bianisotropic medium.

Besides many quasicrystals contain rare-earth.

Components and at low temperatures they may have magnetic structure. In such case their physical properties should be characterized by groups of magnetic symmetry

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# On the wave normal equation for bianisotropic media

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For plane time-harmonic electromagnetic waves  $\mathbf{E}e^{i\varphi}$ ,  $\mathbf{H}e^{i\varphi}$ ,  $\varphi = \omega(\mathbf{m}\mathbf{x}/c - t)$  the Maxwell equations have the form [1,3]

$$\mathbf{D} = -\overset{\times}{\mathbf{m}} \mathbf{H} = -[\mathbf{m}, \mathbf{H}], \quad \mathbf{B} = \overset{\times}{\mathbf{m}} \mathbf{E} = [\mathbf{m}, \mathbf{E}]. \quad (1)$$

Here  $\mathbf{m} = n\mathbf{n}$  is the refraction vector [1,2],  $n$  is the refraction index,  $\mathbf{n}$  is the wave normal ( $\mathbf{n}^2 = 1$ ),  $\overset{\times}{\mathbf{m}}$  stands for the antisymmetric tensor dual to the vector  $\mathbf{m}$ , and  $[\mathbf{m}, \mathbf{H}]$  denotes the vector product<sup>1</sup>. For bianisotropic media with the constitutive equations

$$\mathbf{D} = \epsilon\mathbf{E} + \alpha\mathbf{H}, \quad \mathbf{B} = \mu\mathbf{H} + \beta\mathbf{E}, \quad (2)$$

it follows from (1) that

$$\epsilon\mathbf{E} + (\alpha + \overset{\times}{\mathbf{m}})\mathbf{H} = 0, \quad \mu\mathbf{H} + (\beta - \overset{\times}{\mathbf{m}})\mathbf{E} = 0. \quad (3)$$

Eliminating the magnetic field vector  $\mathbf{H}$  we have the wave normal equation

$$|\epsilon - (\alpha + \overset{\times}{\mathbf{m}})\mu^{-1}(\beta - \overset{\times}{\mathbf{m}})| = 0. \quad (4)$$

For arbitrary tensors  $\epsilon$ ,  $\mu$ ,  $\alpha$ , and  $\beta$ , the evaluation of the determinant (4) results in a rather complicated expression. However, we can simplify the analysis by eliminating the tensor  $\mu$  (or  $\epsilon$ ) with the help of the linear transformation [4,5] of all the vectors:

$$\mathbf{E} = \overline{\mu}_1\mathbf{E}', \quad \mathbf{H} = \overline{\mu}_1\mathbf{H}', \quad \mathbf{m} = \overline{\mu}_1\mathbf{m}' \quad (5)$$

and the tensors

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<sup>1</sup>The scalar product between vectors is denoted without any multiplication sign, by ab. The Gaussian system of physical units is used in the paper. (Editors)

$$\epsilon = \mu_1 \epsilon' \mu_1, \quad \mu = \mu_1 \mu' \mu_1, \quad \alpha = \mu_1 \alpha' \mu_1, \quad \beta = \mu_1 \beta' \mu_1, \quad \overset{\times}{\mathbf{m}} = \mu_1 \overset{\times}{\mathbf{m}}' \mu_1. \quad (6)$$

Here  $\mu_1^2 = \mu$ , and  $\bar{\mu}_1 = |\mu_1| \mu_1^{-1}$  is the tensor adjoint to the tensor  $\mu_1$ . If  $\mu$  is a symmetric and positive definite tensor then  $\mu_1$  always exists and it has the same properties. After such replacement we obtain from (3):

$$\epsilon' \mathbf{E}' + (\alpha' + \overset{\times}{\mathbf{m}}') \mathbf{H}' = 0, \quad \mathbf{H}' + (\beta' - \overset{\times}{\mathbf{m}}') \mathbf{E}' = 0, \quad (7)$$

Note that always

$$(\bar{\mu}_1 \mathbf{m}')^{\times} = \mu_1 \overset{\times}{\mathbf{m}}' \mu_1 \quad (8)$$

since  $\bar{\mu}_1 = \mu_1$  (see [5])<sup>2</sup>. Also,  $\mu' = \mu_1^{-1} \mu \mu_1^{-1} = 1$ .

If  $\mu = 1$  we have from (4)

$$\Delta = |\epsilon - (\alpha + \overset{\times}{\mathbf{m}})(\beta - \overset{\times}{\mathbf{m}})| = |\gamma + \kappa + \overset{\times}{\mathbf{m}}^2| = 0, \quad (9)$$

where

$$\gamma = \epsilon - \alpha\beta, \quad \kappa = \alpha \overset{\times}{\mathbf{m}} - \overset{\times}{\mathbf{m}} \beta. \quad (10)$$

With the help of the covariant methods expounded in [1,3,5] we obtain

$$\Delta = \Delta_0 + \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4, \quad (11)$$

where each term  $\Delta_k$  is a polynomial of  $\mathbf{m}$  of the power  $k$ :

$$\begin{aligned} \Delta_0 &= |\gamma|, \quad \Delta_1 = (\bar{\gamma}\kappa)_t, \quad \Delta_2 = (\bar{\gamma} \overset{\times}{\mathbf{m}}^2 + \gamma \bar{\kappa})_t, \\ \Delta_3 &= |\kappa| + \left( (\bar{\gamma} + \bar{\kappa} - \bar{\gamma} - \bar{\kappa}) \overset{\times}{\mathbf{m}}^2 \right)_t, \quad \Delta_4 = \left( \bar{\kappa} \overset{\times}{\mathbf{m}}^2 + \gamma \bar{\overset{\times}{\mathbf{m}}}^2 \right)_t. \end{aligned} \quad (12)$$

Here the index  $t$  denotes the trace of a tensor. From (10) and (8) we obtain, using the same methods [1,3,5]<sup>3</sup>:

$$\begin{aligned} \Delta_0 &= |\epsilon| - (\bar{\epsilon}\alpha\beta)_t + (\epsilon\bar{\beta}\bar{\alpha})_t - |\alpha\beta|, \\ \Delta_1 &= \left( (\bar{\gamma}\alpha - \beta\bar{\gamma}) \overset{\times}{\mathbf{m}} \right)_t, \quad \Delta_2 = A + B\mathbf{m}^2 + \mathbf{m}\rho\mathbf{m}, \\ A &= (\alpha \overset{\times}{\mathbf{m}})_t \left( \beta(\gamma - \gamma_t) \overset{\times}{\mathbf{m}} \right)_t + (\beta \overset{\times}{\mathbf{m}})_t (\gamma\alpha \overset{\times}{\mathbf{m}})_t, \quad B = (\alpha\beta(\gamma - \gamma_t))_t - \bar{\gamma}_t, \end{aligned} \quad (13)$$

<sup>2</sup> $\bar{\alpha}$  denotes the transpose of a matrix. (Editors)

<sup>3</sup>Here and in the following the unity matrix is omitted, so that  $\gamma - \gamma_t$  means the matrix  $\gamma$  minus its trace multiplied by the unity matrix. Unlike in the Western tradition, here  $\mathbf{a}.\mathbf{b}$  denotes the dyadic product between vectors. In the expression for  $\Delta_4$  we have in fact the product of two scalars. (Editors)

$$\begin{aligned}\rho &= \bar{\gamma} + \bar{\alpha}\gamma + \gamma\bar{\beta} - \beta(\gamma - \gamma_t)\alpha - \bar{\gamma}^{-1} \left( \beta\alpha\bar{\gamma} - (\beta\alpha\bar{\gamma})_t \right), \\ \Delta_3 &= \mathbf{m}\gamma\mathbf{m} \left( (\beta - \alpha) \overset{\times}{\mathbf{m}} \right)_t + [\mathbf{m}, \gamma\mathbf{m}](\bar{\alpha} + \beta)\mathbf{m} - [\mathbf{m}, \bar{\alpha}\mathbf{m}](\bar{\alpha} + \beta - \beta_t)\beta\mathbf{m}, \\ \Delta_4 &= \mathbf{m}^2\mathbf{m}\epsilon\mathbf{m} - \mathbf{m}\alpha\mathbf{m}\cdot\mathbf{m}\beta\mathbf{m}.\end{aligned}$$

If  $\alpha = \beta = 0$ , the equations (11), (13) simplify to

$$\mathbf{m}^2\mathbf{m}\epsilon\mathbf{m} + \left( \bar{\epsilon} \overset{\times}{\mathbf{m}}^2 \right)_t + |\epsilon| = 0, \quad (14)$$

which coincides with the covariant wave normal equation obtained in [1,3]. With the help of the transformation inverse to that in (5), (6) ( $\mathbf{m} \rightarrow \bar{\mu}_1^{-1}\mathbf{m}$ ,  $\epsilon \rightarrow \mu_1^{-1}\epsilon\mu_1^{-1}$ ) we obtain from here the corresponding equation for magneto-anisotropic media  $\mathbf{m}\epsilon\mathbf{m}\cdot\mathbf{m}\mu\mathbf{m} + \mathbf{m}\mu(\bar{\epsilon}\mu - (\bar{\epsilon}\mu)_t)\mathbf{m} + |\epsilon\mu| = 0$ , first deduced in [1,7] (see also [3,5]).

In the case  $\Delta_1 \neq 0$ ,  $\Delta_3 \neq 0$  the medium is nonreciprocal. If  $\beta = -\bar{\alpha}$  we have from (13)  $\Delta_1 = \Delta_3 = 0$ , i.e. this condition connecting  $\alpha$  and  $\beta$  is sufficient for reciprocity in the general case [8]. Under this condition,  $\gamma = \epsilon + \alpha\bar{\alpha} = \bar{\gamma}$  and we have from (13) that

$$\begin{aligned}\Delta_0 &= |\epsilon| + (\bar{\epsilon}\alpha\bar{\alpha})_t + (\bar{\epsilon}\bar{\alpha}\bar{\alpha})_t + |\alpha|^2, \\ A &= (\alpha \overset{\times}{\mathbf{m}})_t \left( (2\gamma - \gamma_t)\alpha \overset{\times}{\mathbf{m}} \right)_t, \quad B = (\bar{\alpha}\bar{\alpha})_t - \bar{\epsilon}_t - 2((\epsilon - \epsilon_t)\alpha\bar{\alpha})_t, \\ \rho &= \bar{\gamma} + \bar{\alpha}\gamma + \gamma\bar{\alpha} + \bar{\alpha}(\gamma - \gamma_t)\alpha + \gamma^{-1}(\bar{\alpha}\alpha\bar{\gamma} - (\bar{\alpha}\alpha\bar{\gamma})_t) = \bar{\rho}, \\ \Delta_4 &= \mathbf{m}^2\mathbf{m}\epsilon\mathbf{m} + (\mathbf{m}\alpha\mathbf{m})^2.\end{aligned} \quad (15)$$

However, the condition  $\beta = -\bar{\alpha}$  is not necessary as we can see from an example of uniaxial bianisotropic medium. For such media  $\epsilon = \epsilon_0 + \epsilon_1\mathbf{e}\cdot\mathbf{e}$  and

$$\alpha = a_0 + a_1\mathbf{e}\cdot\mathbf{e} + a_2 \overset{\times}{\mathbf{e}}, \quad \beta = b_0 + b_1\mathbf{e}\cdot\mathbf{e} + b_2 \overset{\times}{\mathbf{e}}, \quad (16)$$

where  $\mathbf{e}$  is the unit vector ( $\mathbf{e}^2 = 1$ ) along the symmetry axis. The multitude of tensors of the shape (16) forms a commutative algebra. Its elements are given by three numbers:  $\alpha = (a_0, a_1, a_2)$ . The law of multiplication is

$$\alpha\beta = (a_0, a_1, a_2)(b_0, b_1, b_2) = (a_0b_0 - a_2b_2, a_0b_1 + a_1b_0 + a_1b_1 + a_2b_2, a_0b_2 + a_2b_0) = \beta\alpha. \quad (17)$$

We have  $|\alpha| = (a_0 + a_1)(a_0^2 + a_2^2)$ ,  $\bar{\alpha} = (a_0(a_0 + a_1), a_2^2 - a_0a_1, -a_2(a_0 + a_1))$ , and

$$\alpha^{-1} = (a_0, a_1, a_2)^{-1} = \frac{1}{a_0^2 + a_2^2} \left( a_0, \frac{a_2^2 - a_0a_1}{a_0 + a_1}, -a_2 \right). \quad (18)$$

These relations allow us to evaluate all the expressions in (13). Denoting  $\gamma = (g_0, g_1, g_2)$ ,  $\bar{\gamma} = (g'_0, g'_1, g'_2)$ ,  $(\bar{\alpha} + \beta - \beta_t)\beta = (c_0, c_1, c_2)$  we obtain

$$\Delta_1 = 2D_0 \mathbf{em}, \quad \Delta_3 = \mathbf{em} (D_1 \mathbf{m}^2 + D_2 (\mathbf{em})^2), \quad (19)$$

$$D_0 = g'_0(b_2 - a_2) + g'_2(b_0 - a_0), \quad D_1 = 2g_0(b_2 - a_2) + K, \\ D_2 = 2g_1(b_2 - a_2) - K, \quad K = g_1(a_2 - b_2) + g_2(a_1 + b_1) - a_1 c_2 - a_2 c_1, \quad (20)$$

It is easy to see that for uniaxial media the expressions (13) contain the wave normal vector  $\mathbf{n}$  only in the form  $\mathbf{en} = \cos \theta$ , therefore the refraction index  $n$  depends only on the angle  $\theta$  between the propagation direction and the symmetry axis. For any direction of  $\mathbf{n}$  orthogonal to the axis  $\mathbf{e}$  ( $\mathbf{en} = 0$ ) the reciprocity condition holds (see (19)). When  $\alpha$  and  $\beta$  are independent and  $\alpha = \tilde{\alpha}$ ,  $\beta = \tilde{\beta}$  ( $a_2 = b_2 = 0$ ) all the coefficients with the index 2 are equal to zero and also  $D_0 = D_1 = D_2 = \Delta_1 = \Delta_3 = 0$ . So the condition  $\beta = -\tilde{\alpha}$  is not necessary for reciprocity. All these conclusions are true also when  $\mu = \mu_0 + \mu'_0 \mathbf{e.e}$ , i.e. for any uniaxial medium.

General conditions for reciprocity of uniaxial media read  $D_0 = D_1 = D_2 = 0$ . Since we have 8 coefficients in  $\epsilon$ ,  $\alpha, \beta$  (or 10 if  $\mu \neq 1$ ) then evidently these three conditions can be satisfied by a wide spectrum of parameters. In particular, from  $D_1 = D_2 = 0$  we have  $(g_0 + g_1)(b_2 - a_2) = 0$ . Therefore, in addition to the conditions  $\beta = -\tilde{\alpha}$  or  $\alpha = \tilde{\alpha}$ ,  $\beta = \tilde{\beta}$ , which concern only the tensors  $\alpha$  and  $\beta$ , there also exist reciprocity conditions related to the equation  $g_0 + g_1 = 0$ . These affect  $\epsilon_0$  and  $\epsilon_1$ .

Evidently we can analyse any other special cases based on the general relations (11)–(20).

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# THE DETERMINATION OF OPTICAL ANISOTROPIC PARAMETERS OF ABSORBING GYROTROPIC MEDIA

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Anisotropic parameters such as birefringence, dichroism and optical activity parameters (gyrotropic properties) are main characteristics of substances. In [1,2] the methods are offered of the simultaneous determination of anisotropic parameters. We develop these methods for the biaxial absorbing gyrotropic crystals, in which the eigen waves are nonorthogonal (angle of nonorthogonality is  $\theta$ ) and ones have different ellipticities ( $k_1$  and  $k_2$ ). These parameters can be determined from measurements either of the azimuth or of the intensity of the passed light. Here we consider only the second case. We propose to use measurements of the light intensity depending on the rotation angle  $\alpha$  of the investigated plate placed between arbitrary oriented polarizers.

We have received the expression for the intensity by multiplying the Mueller matrices of the analyzer, the investigating plate and the polarizer:

$$I(\alpha) = M (a + b_1 \cos 2\alpha + b_2 \sin 2\alpha + c_1 \cos 4\alpha + c_2 \sin 4\alpha) \quad (1)$$

where  $a, b_{1,2}, c_{1,2}$  depend on values:  $\Delta = 2\pi d(n_2 - n_1)/\lambda$ ,  $\delta = 2\pi d(\epsilon_2 - \epsilon_1)/\lambda$ , ellipticities  $k_{1,2} = \tan \gamma_{1,2}$  and azimuths  $\chi_1, \chi_2 = \chi_1 + (\pi/2 - \theta)$  of eigen wave,  $(n_2 - n_1)$ ,  $(\epsilon_2 - \epsilon_1)$  are birefringence and dichroism of eigen waves respectively,  $d$  is the thickness of investigating plate,  $\lambda$  is the length wave of the incident light. The function  $I(\alpha)$  is given by in Figure for two cases: polarizers are crossed ( $I^\perp(\alpha)$ ) and ones are parallel ( $I^\parallel(\alpha)$ ). It is seen that at crossed polarizers at  $\theta \neq 0$  and  $k_1 \neq k_2$  both minima and maxima are different and the all dependence  $I^\perp(\alpha)$  is raised on abscissa axis. For arbitrary sings of  $k_1$  and  $k_2$  the  $I^\perp(\alpha)$  form are always the same. If polarizers are parallel the  $I^\parallel(\alpha)$  form are different in depending on sings of  $k_1, k_2$ .

At crossed polarizers the form of  $I^\perp(\alpha)$  depends on parameters  $\theta$  and  $\gamma_{1,2}$  whereas parameters  $\Delta$  and  $\delta$  influence only at the scale. Therefore we can determine values  $\theta$  and  $k_{1,2}$  from measurements  $a^\perp, b_i^\perp, c_i^\perp$  at crossed polarizers. At first an nonorthogonality angle  $\theta$  is

calculated from equation:

$$\cos^3 2\theta + (a^\perp/c^\perp) \cos^2 2\theta - [1 - (b^\perp/2c^\perp)^2] \cos 2\theta + \\ + [[(b_1^\perp)^2 - (b_2^\perp)^2]c_1^\perp + 2b_1^\perp b_2^\perp c_2^\perp - 4a^\perp c^\perp]/4(c^\perp)^3 = 0, \quad (2)$$

where  $(b^\perp)^2 = (b_1^\perp)^2 + (b_2^\perp)^2$ ,  $(c^\perp)^2 = (c_1^\perp)^2 + (c_2^\perp)^2$ .

By taking the ratio  $c_2^\perp/c_1^\perp$  one can determine the origin azimuth of the plate

$$\operatorname{tg} \alpha_0 = (c_1^\perp \operatorname{tg} 2\theta + c_2^\perp) / (c_2^\perp \operatorname{tg} 2\theta - c_1^\perp). \quad (3)$$

By taking the ratio  $a^\perp/c^\perp$  and  $(b^\perp)^2/(2c^\perp)^2$  we receive the system of two equations. By solving it we obtain expressions for ellipticities of eigen waves

$$k_{1,2}^2 = \frac{\sqrt{(b^\perp)^2 + q \cos^2 \theta} \pm \sqrt{(b^\perp)^2 - q \sin^2 \theta} - 4c^\perp}{\sqrt{(b^\perp)^2 + q \cos^2 \theta} \pm \sqrt{(b^\perp)^2 - q \sin^2 \theta} + 4c^\perp}, \quad (4)$$

where  $q = 8c^\perp (a^\perp + c^\perp \cos 2\theta)$ .

It is evident that sines of  $k_1$  and  $k_2$  from measurement  $I^\perp(\alpha)$  are not determined.

Thus we calculated  $\theta$ ,  $\alpha_0$  and  $k_{1,2}$  using measurements  $I^\perp(\alpha)$  at crossed polarizers. Then we can determine parameters  $\Delta$  and  $\delta$  using measurements  $I^\parallel(\alpha)$  at parallel polarizers. At first from  $b_1^\parallel/2c^\parallel$ ,  $b_2^\parallel/2c^\parallel$ ,  $a^\parallel/c^\parallel$  one calculates value  $\delta$ :

$$\exp(2\delta) = \frac{(Z_1 + Z_2)[a^\parallel p - c^\parallel(Z_1 Z_2 + R_1 R_2)] + p[b_1^\parallel(X_1 Z_2 - X_2 Z_1) + b_2^\parallel(Y_1 Z_2 - Y_2 Z_1)]}{(Z_1 + Z_2)[a^\parallel p - c^\parallel(Z_1 Z_2 + R_1 R_2)] - p[b_1^\parallel(X_1 Z_2 - X_2 Z_1) + b_2^\parallel(Y_1 Z_2 - Y_2 Z_1)]}, \quad (5)$$

where  $p = \cos 2\gamma_1 \cos 2\gamma_2$ ;  $R_1 R_2 = X_1 X_2 + Y_1 Y_2 + Z_1 Z_2$ ;

$X_i = \cos 2\chi_i \cos 2\gamma_i$ ;  $Y_i = \sin 2\chi_i \cos 2\gamma_i$ ;  $Z_i = \sin 2\gamma_i$  ( $i=1,2$ ).

At second from the same relations one can calculate value  $\Delta$ :

$$\cos \Delta = \frac{\exp(2\delta) + 1}{2\exp(\delta)} \cdot \frac{(a^\parallel - c^\parallel \cos 2\theta) \cos 2\gamma_1 \cos 2\gamma_2 - 2c^\parallel}{(a^\parallel + c^\parallel \cos 2\theta) \cos 2\gamma_1 \cos 2\gamma_2 - 2c^\parallel \sin 2\gamma_1 \sin 2\gamma_2}. \quad (6)$$

In the case  $k_1 = k_2 = 0$  and  $\theta = 0$  we obtain simple relationships for the calculating  $\delta$  and  $\Delta$  from (5) and (6):

$$\exp(2\delta) = (a^\parallel + b^\parallel + c^\parallel) / (a^\parallel - b^\parallel + c^\parallel) \\ \cos \Delta = (a^\parallel - 3c^\parallel) / \sqrt{(a^\parallel + c^\parallel)^2 - (b^\parallel)^2}. \quad (7)$$

By knowing parameters  $\delta$  and  $\Delta$  we can calculate dichroism ( $\mathcal{D}_2 - \mathcal{D}_1$ ) and birefringence ( $n_2 - n_1$ ) of the investigating plate.

Hence we can determine all optical anizotropic parameters of the absorbing gyrotropic crystal from measurements of light intensity  $I^\perp(\alpha)$  and  $I^\parallel(\alpha)$ . Such kind of dependences may be obtained on spectrophotometer if one places the sample between polarizers and one measures the intensity  $I(\alpha)$  when turning the sample by an angle  $\alpha$ . We built up the polarization equipment for spectrophotometer which consists of the polarizer and the analyzer in special setting with angle limbs and the same setting for investigating samples. The special program have make for the calculating of coefficients  $a^\perp$ ,  $b^\perp_i$ ,  $c^\perp_i$  and  $a^\parallel$ ,  $b^\parallel_i$ ,  $c^\parallel_i$  and then birefringence, dichroism, ellipticities and the nonorthogonality angle of eigen waves.

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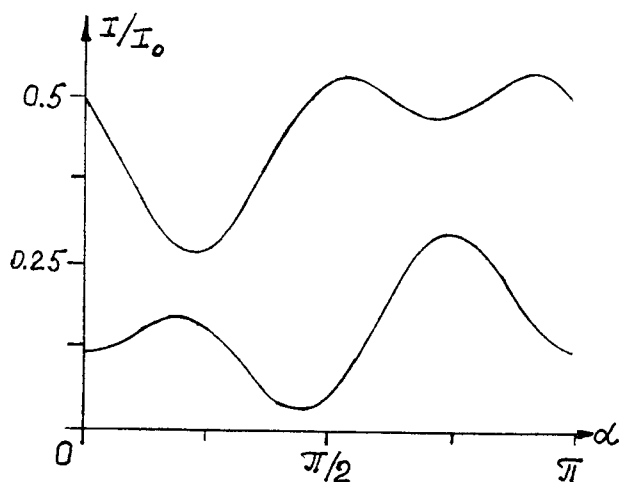


Figure. Dependence of light intensity  $I(\alpha)$  on a plate rotation angle  $\alpha$  at values:  $\alpha_0 = 0$ ,  $\theta = 10^\circ$ ,  $k_1 = -0.4$ ,  $k_2 = 0.2$ ,  $\Delta = 75^\circ$ ,  $\delta = 0.1$ . Up - polarizers are parallel; down - polarizers are crossed.



# Non-collinear interaction of electromagnetic waves in gyrotropic crystals with rapidly rotating anisotropy

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The interaction of electromagnetic waves (in the microwave and infrared frequency bands) with electric fields rapidly rotating in time and forming rotating helical anisotropy in non-linear gyrotropic crystals is investigated in our research. Such effects as selective reflection and amplification of a test signal with the frequency and direction of polarization vector coinciding with those of the induced anisotropy had been demonstrated in our previous works in a case of the test and modulating waves propagating normally to the crystal surface [1,2]. Thus such crystals possess properties characteristic to cholesteric liquid crystals (CLC). At the same time they allow one to realize electromagnetic waves parametric interaction causing resonance amplification (or attenuation) of the test electromagnetic wave.

The present paper is devoted to the research of test electromagnetic wave propagation in a case of oblique incidence onto the crystal with rapidly rotating helicoidal anisotropy. Such anisotropy is formed in non-linear gyrotropic crystal by two intensive light waves of right-handed and left-handed circular polarizations and different frequencies. Their wave-number difference makes a significant contribution to the crystal gyrotropy. The induced anisotropy can be described by the effective permittivity tensor

$$\epsilon(z, t) = U(z, t)\epsilon U^{-1}(z, t), \quad (1)$$

Where  $U(z, t)$  is the rotation operator relatively to the  $z$ -axis [3] coinciding with the modulating waves propagation direction,  $\epsilon$  is the local and momentary permittivity tensor that is optically uniaxial.

Non-collinear interaction between modulating and test waves has some specific features: according to the diffraction theory, the higher reflection orders appear at the frequencies divisible by the Bragg's one. Moreover, as it was mentioned in CLC-optics publications, even in crystals with stationary spiral anisotropy the reflection character changes essentially already for the first-order diffraction: polarization properties of the eigenmodes become complicated and bands of any polarized light reflection appear [4].

The solution of the first-order diffraction problem has been obtained in the present work with the help of the Slowly Changing Amplitudes method under the two-waves approximation. The electric field vector is assumed in the diffraction theory to be expressed as a sum of the  $\pi$ - and  $\sigma$ -components:

$$\mathbf{E}_0 = (E_{0\sigma}\boldsymbol{\sigma} + E_{0\pi}\boldsymbol{\pi}) \exp[i(k_0(\omega_0)r - \omega_0 t)], \quad (2)$$

which are parallel and orthogonal to the incidence plane, respectively. Taking into account the problem geometry, the Bragg's condition  $\mathbf{k}_1 = \mathbf{k}_0 + \boldsymbol{\tau}$  acquires the form

$$\omega_0 \sin \theta_0 + (\omega_0 - 2\Delta\Omega) \sin \theta_1 = 2\Delta K c \sqrt{\bar{\epsilon}}. \quad (3)$$

Here  $\mathbf{k}_1$  is the propagation vector of the reflected wave,  $\boldsymbol{\tau}$  is the reciprocal lattice vector of the induced structure,  $\Delta\Omega$  and  $\Delta K$  are the modulating waves frequency and wavenumbers differences,  $c$  is the light velocity (the angles  $\theta_0$  and  $\theta_1$  are shown in Figure 1)<sup>1</sup>

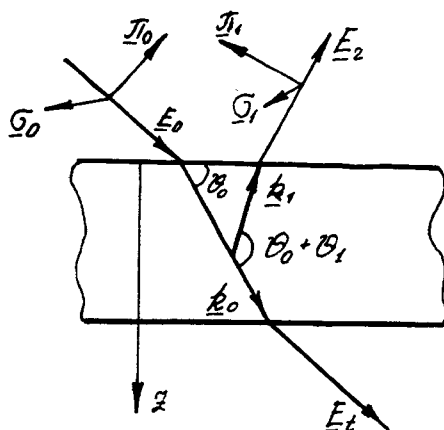


Figure 1. The geometry of the problem.

The amplitudes of the  $\sigma$ - and  $\pi$ -components are the solutions of the differential equations system which may be represented as

$$\frac{d}{dz} \begin{pmatrix} E_{0\sigma} \\ E_{0\pi} \\ E_{1\sigma} \\ E_{1\pi} \end{pmatrix} = \hat{\mathcal{A}} \begin{pmatrix} E_{0\sigma} \\ E_{0\pi} \\ E_{1\sigma} \\ E_{1\pi} \end{pmatrix}. \quad (4)$$

The matrix  $\mathcal{A}$  is determined by the crystal and interacting waves parameters as well as by the interaction geometry, the subindices 1 and 0 mark the incident and reflected

<sup>1</sup> $\bar{\epsilon} = \epsilon_0 + 2\theta E_0^2$ , where  $\theta$  is the coefficient of electro-optical interaction. (Editors)

waves, correspondingly. Solution of dispersion equation which results from solvability condition of the system (4) gives an opportunity to determine the wavenumbers of the incident and reflected waves for the case of the test wave propagation obliquely to the induced helix anisotropy axis. The results obtained were used to solve the boundary problem. Boundary conditions were selected in analogy with the reflecting hologram problems:

$$E_{1\sigma}(L) = 0, \quad E_{1\pi}(L) = 0, \quad E_{0\sigma}(0) + E_{0\pi}(0) = E_0, \quad (5)$$

where  $L$  is the crystal thickness.

Boundary problem solution has been obtained by numerical methods using a personal computer. It has been demonstrated that reflection of arbitrary polarized waves is possible in a case of oblique incidence of the test wave on the surface of a crystal with induced helical rotating anisotropy. Simultaneous amplification (or attenuation) of both the transformed and reflected waves is possible in this case, in the contrary to that of the normal incidence.

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# Microwave effective permeability of conductive helices

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Interest in bianisotropic materials has been recently widely increased since they offer some novel promising applications not only in optical technology, but in microwave technology and radio engineering, too. There are a lot of theoretical articles dealing with various bianisotropic media [1-2]. Nevertheless there are only few works in the field of the experimental investigation of these materials in the microwave region, especially there are no experimental studies of influence of individual inclusions on the properties of such materials. The main purpose of this work was to investigate effective permeability of wire helices in the region of centimeter wavelengths.

A resonator method with transmittance measurement is used to determine the effective complex permeability of wire helices. We used a cylindrical resonator with a high  $Q$ -factor and with a possibility to vary its resonance frequency in the region from 2.8 GHz to 6 GHz in the experiment. The samples under investigation were helices with characteristic sizes  $d, l \ll \lambda$  ( $d$  is the helix diameter,  $l$  stands for the length of the helix, and  $\lambda$  is the wavelength). These samples were placed into the maximum of magnetic field amplitude (high  $Q$ -factor  $H_{011}$  mode was utilized) at the axis of the resonator. The axis of the helix was parallel to the magnetic field vector. In order to calculate the effective complex permeability of a sample, we measured the resonance frequencies and  $Q$ -factors of empty ( $f_0, Q_0$ ) and loaded ( $f_1, Q_1$ ) resonators. After that we calculated the effective magnetic moment of a sample using the perturbation technique [3] and then we carried out the normalization of the data obtained. As a factor of normalization we took the total volume of the sample ( $V = \pi d^2 l / 4$ ). Formulas for calculation of the effective complex permeability  $\hat{\mu} = \mu' - j\mu''$  were as follows:

$$\mu' = 1 + \frac{1}{h} \frac{f_0 - f_1}{f_0}$$
$$\mu'' = \frac{2}{h} \left( \frac{1}{Q_1} - \frac{1}{Q_0} \right),$$

where  $h$  is the form-factor. This value equals to the ratio of the energy in the volume of the sample divided by the energy in the whole resonator. The results obtained were tested by measuring a sample of iron powder which had the same shape. The

permeability of this sample measured in resonator was in good agreement with results obtained by another experimental method.

It is worth noting that the chirality factor had no influence on measurements of  $\hat{\mu}$  of helices, because in this case the electric field at the axis of the resonator equals zero, the electric field in the volume of the sample was negligible and thus the chirality factor of the helix did not perturb the electromagnetic field in resonator [4].

The helices under investigation were 3 mm in diameter and had the pitch equal to 1 mm. These samples were made by use of wires with various conductances and thicknesses. The results of measurement of effective values of  $\hat{\mu} = \mu' - j\mu''$  for three different samples are presented in Fig. 1-3. The dependence of  $\mu'$  and  $\mu''$  on the frequency has the resonance character. The real part of the permeability may take on values both greater than unity ( $\mu' > 1$  — the paramagnetic effect) and smaller than unity ( $\mu' < 1$  — the diamagnetic effect). The losses reach the maximum value at the resonance:  $\mu'' = \max$ , and  $\mu'$  is zero. It is typically that the predicted resonance frequency is in disagreement with the lengthwise resonance which one should obtain at a particular wire's length of helix, as  $2 * L/\lambda_0 \approx 0.6 - 0.7$  (where  $L$  is the wire's length, and  $\lambda_0$  is the resonant wavelength).

The analysis of the presented curves shows that when the resistance of the wire increases then the resonance values of  $\mu'$  and  $\mu''$  decrease, but at the same time the resonance frequency band width  $\Delta f$  becomes wider. As a result the product  $\mu''_{\max} \Delta f$  is approximately constant.

Thus, it is shown that conductive helices with certain sizes  $d, l \ll \lambda$  have a strong resonance in microwave region. They have large magnetic losses in the resonance region and, in addition, the helices under investigation have paramagnetic properties in some frequency bands.

## References

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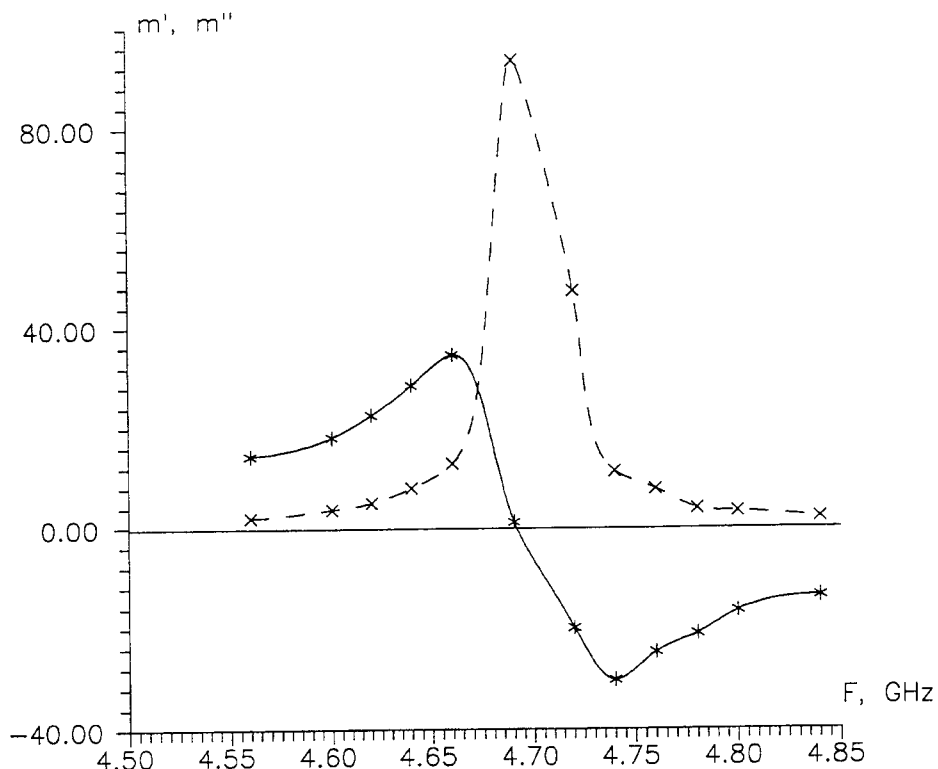


Fig.1. The dependence of  $m'$  (solid line) and  $m''$  (dashed line) on frequency of copper helix. The thickness of wire is 70 mkm. The helix has 2,5 turns.

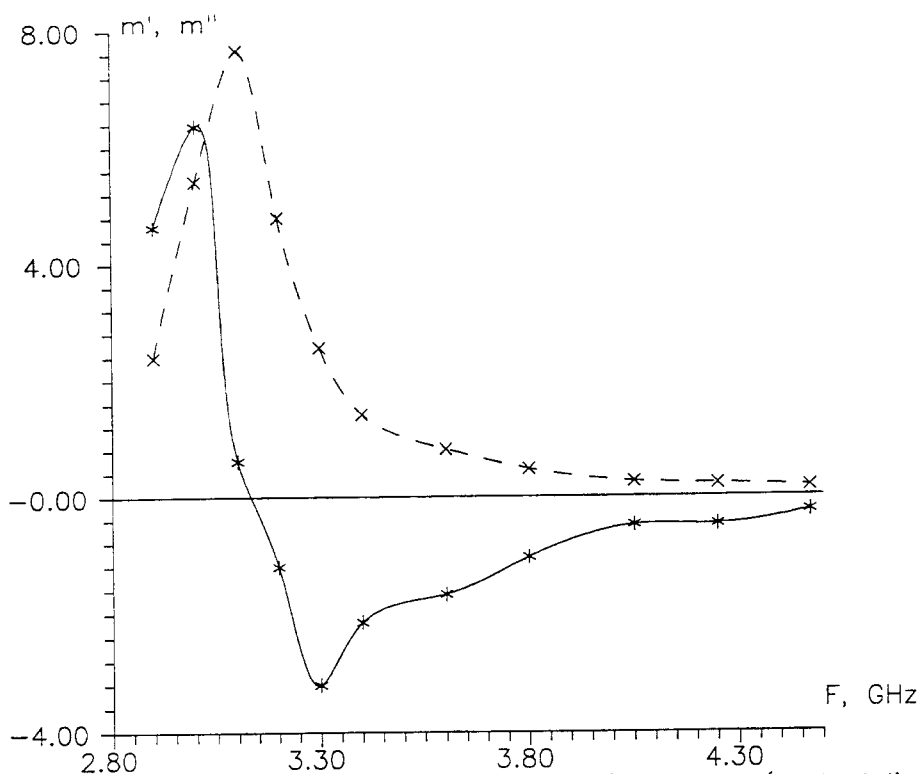


Fig.2. The dependence of  $m'$  (solid line) and  $m''$  (dashed line) on frequency of nickel-chromium alloy wire helix. The thickness of the wire is 30 mkm. The helix has 3,75 turns.

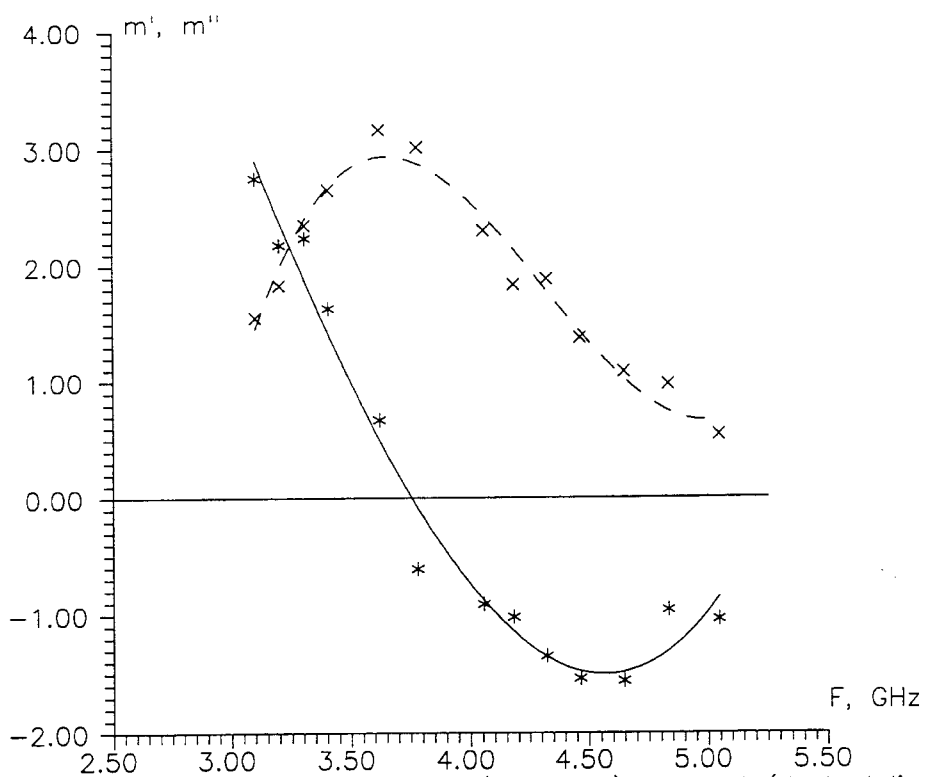


Fig.3. The dependence of  $m'$  (solid line) and  $m''$  (dashed line) on frequency of carbon helix. The thickness of wire is 300 mkmm. The helix has 2,75 turns.